Retrofitting and strengthening the ML module system

PhD Defense *- 11/12/24*

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Under the supervision of Didier Rémy (Inria) and Gabriel Radanne (Inria)

Disc
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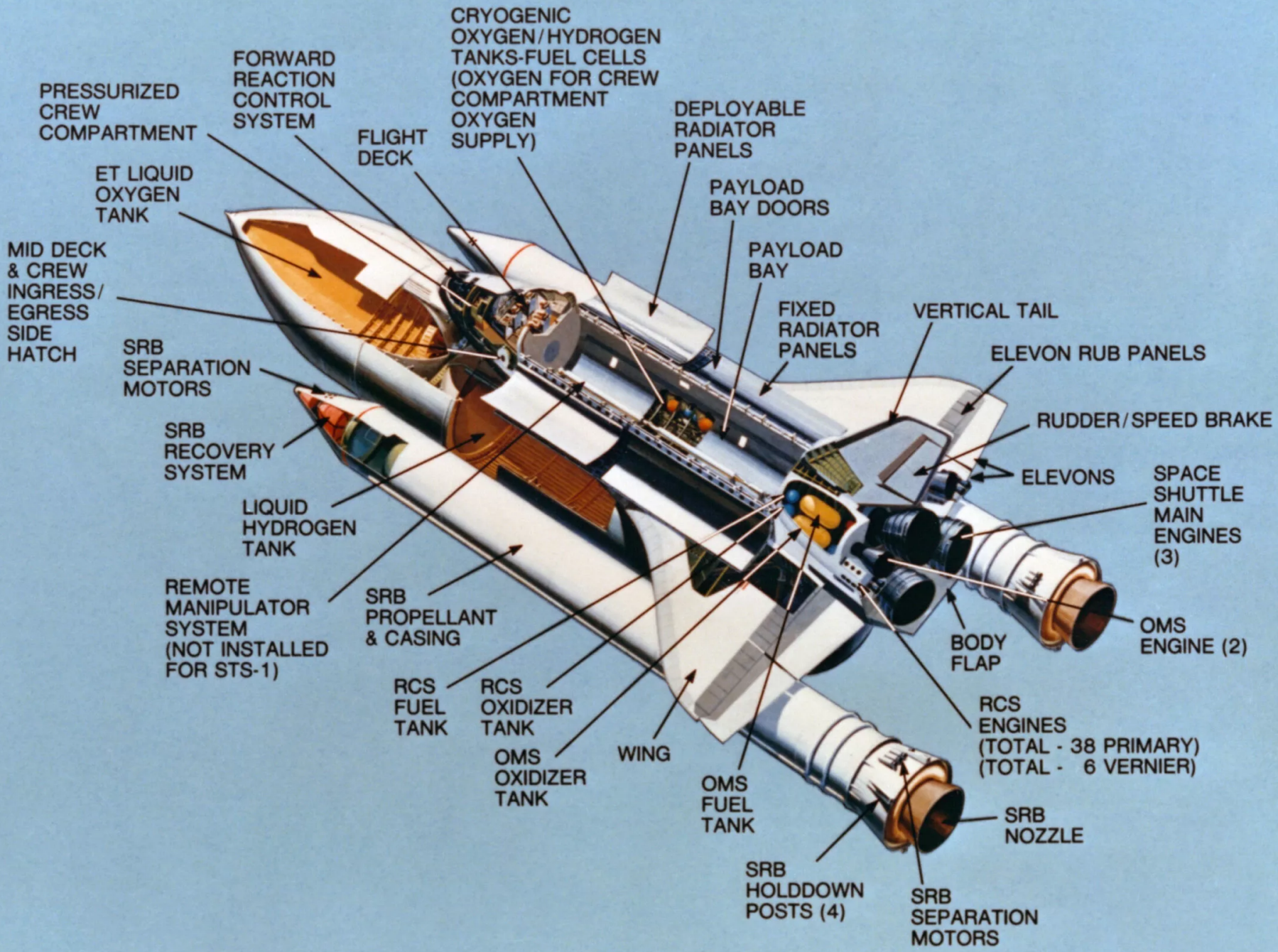
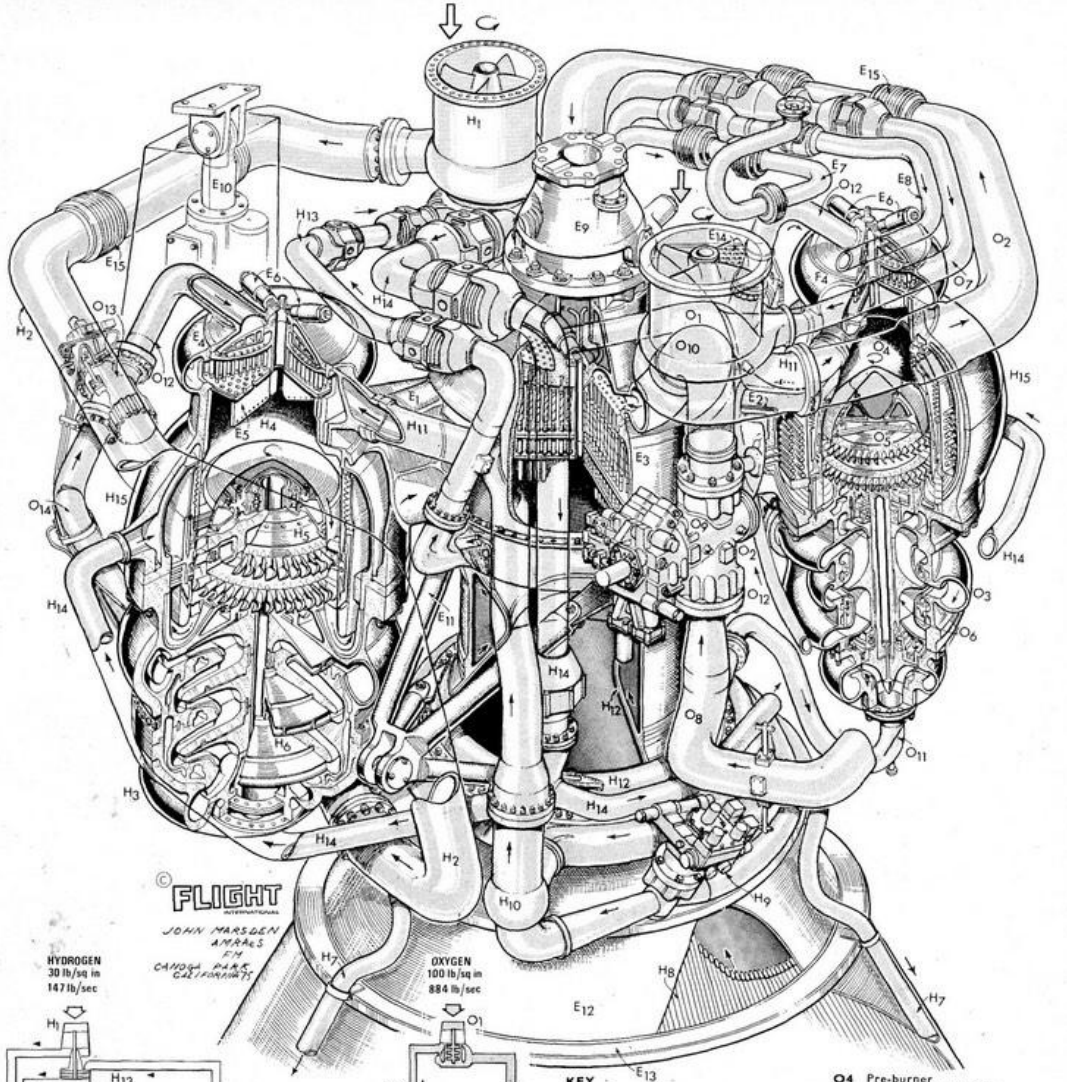
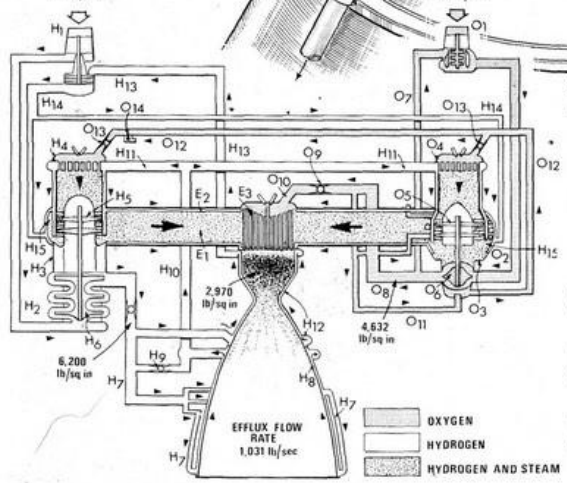

PRESSURIZED CREW COMPARTMENT
FORWARD REACTION CONTROL SYSTEM
CRYOGENIC OXYGEN/HYDROGEN TANKS-FUEL CELLS (OXYGEN FOR CREW COMPARTMENT OXYGEN SUPPLY)
FLIGHT DECK
DEPLOYABLE RADIATOR PANELS
ET LIQUID OXYGEN TANK
PAYLOAD BAY DOORS
PAYLOAD BAY
MID DECK & CREW INGRESS/ EGRESS SIDE HATCH
FIXED RADIATOR PANELS
VERTICAL TAIL
ELEVON RUB PANELS
SRB SEPARATION MOTORS
RUDDER/SPEED BRAKE
SRB RECOVERY SYSTEM
ELEVONS
SPACE SHUTTLE MAIN ENGINES (3)
LIQUID HYDROGEN TANK
REMOTE MANIPULATOR SYSTEM (NOT INSTALLED FOR STS-1)
SRB PROPELLANT & CASING
BODY FLAP
OMS ENGINE (2)
RCS FUEL TANK
RCS OXIDIZER TANK
RCS ENGINES (TOTAL - 38 PRIMARY) (TOTAL - 6 VERNIER)
OMS OXIDIZER TANK
WING
OMS FUEL TANK
SRB NOZZLE
SRB HOLDDOWN POSTS (4)
SRB SEPARATION MOTORS





KEY

Hydrogen system

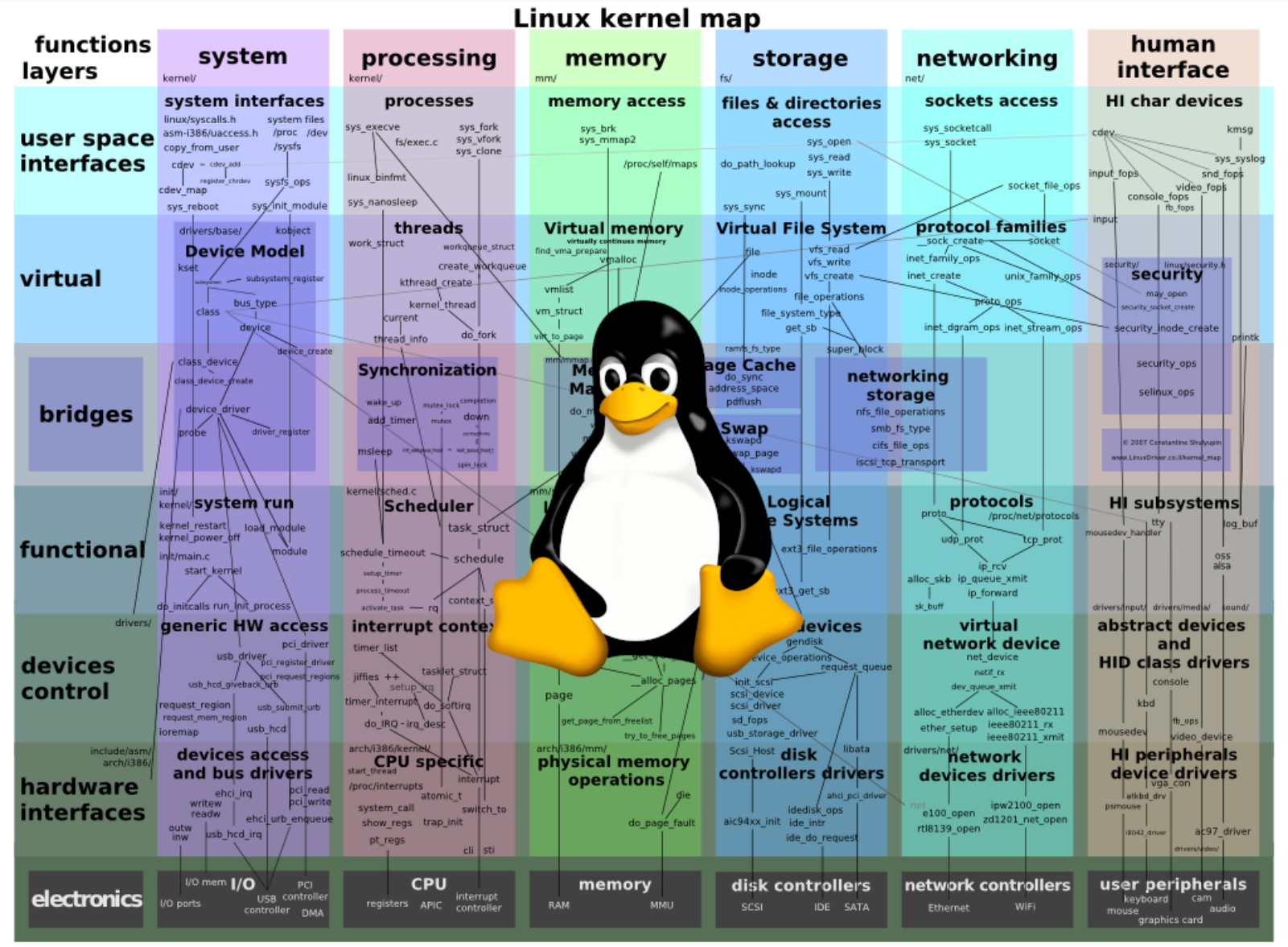
- **H1** Low-pressure turbopump
- **H2** Fuel delivery pipe from l-p pump to high-pressure turbopump
- **H3** High-pressure turbopump
- **H4** Pre-burner
- **H5** High-pressure turbine
- **H6** High-pressure centrifugal compressor
- **H7** Nozzle-coolant supply pipes
- **H8** Nozzle thrust chamber showing tapered tube assembly providing regenerative cooling circuit
- **H9** Coolant bypass control valve
- **H10** Pre-burner supply pipe
- **H11** Pre-burner supply pipe, bifurcated to supply oxygen h-p turbopump pre-burner
- **H12** Main combustion chamber manifold (regeneratively cooled)
- **H13** Delivery pipe to l-p turbopump
- **H14** Coolant delivery pipe from l-p turbine to h-p hot-gas manifold
- **H15** Heat-exchanger

Oxygen system

- **O1** Low-pressure turbopump
- **O2** Fuel delivery pipe from l-p pump to high-pressure turbopump
- **O3** High-pressure turbopump
- **O4** Pre-burner
- **O5** High-pressure turbine
- **O6** High-pressure centrifugal compressor
- **O7** Low-pressure turbine drive pipe
- **O8** High-pressure delivery pipe to combustion chamber
- **O9** Main oxidizer metering valve
- **O10** Delivery pipe to main injector
- **O11** Pre-burner turbopump supply pipe
- **O12** Pre-burner oxygen-supply pipe
- **O13** Pre-burner fuel-control valve
- **O14** Oxidizer bleed valve

Engine ancillaries

- **E1** Hot-gas manifold
- **E2** Internal cooling liner
- **E3** Main dome/injector and baffle elements (total 600)
- **E4** Pre-burner co-axial injector elements
- **E5** Baffle plates (three)
- **E6** Dual spark igniters
- **E7** Fuel bleed pipe
- **E8** Oxygen tank pressurisation pipe
- **E9** Gimbal-bearing thrust housing
- **E10** Gimbal actuator (total two)
- **E12** Gimbal-actuator lower support strut
- **E12** External insulation heat jacket
- **E13** Reinforcing "hat-bands"
- **E14** Solid-state control unit
- **E15** Flexible bellows



~30 million lines of code

Handling complex system

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Modularity

- Structure: gather components into *modules* with limited interactions
- Re-usability: Identify common parts reused throughout the system

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- Interfaces
- Abstraction
- Name-spaces
- Composition

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In programming languages

- Functions
- Code/Interfaces files
- Libraries
- Package-managers

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- OOP (C++, Java, Scala, Python, etc.)
- Type-classes (Haskell, Rust, etc.)
- Module systems (OCaml, SML, Coq, etc.)

Plan

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1. **The ML[1] module system**
   - Structure, signatures, functors
   - The signature avoidance problem

[1] *Meta-Language*, not machine learning

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   - Type system *by translation* to F$^\omega$
   - Transparent existentials
   - OOPSLA 2024 - *Fulfilling OCaml modules with transparency*

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   - OOPSLA 2024 - *Fulfilling OCaml modules with transparency*
3. **ZipML**
   - Type system based on *paths*
   - Zipper signatures
   - POPL 2025 - *Avoiding signature avoidance in ML modules with zippers*

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The ML module system

ML modularity

Building structures

```
module Complex =  struct
  type t = float * float (* polar *)
  let zero = (0., 0.)
  let one = (1., 0.)
  let add c c' = ...
  ...
end
```

ML modularity

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let c_two =
 Complex.add Complex.one Complex.one
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val c_two : Complex.t
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end
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```
let c_invalid =
 Complex.add (Complex.one) (-1., 0.)
```

```
(-1,0) is not of type Complex.t
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```
module Polynomials =
  functor (R: Ring) → struct
    type t = R.t list
    let zero = []
    let one = [R.one]
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Composition

```
module CX = Polynomials(Complex)
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module CX : sig ... end
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→ Name spaces

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module CXY =
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→ Name spaces

→ Interface control

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→ Abstraction

ML modularity

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→ Abstraction

→ Parameterization

What flavor for your functor ?

Applicative

Stateless, *pure* modules, weak abstraction barrier → same applications produce equal abstract types

What flavor for your functor ?

Applicative

Stateless, *pure* modules, weak abstraction barrier ⟶ same applications produce equal abstract types

```
module Set(E: Comparable) = (... : sig type t ... end)
module S1 = Set(Complex)
module S2 = Set(Complex)
```

```
let f: S1.t ⟶ S2.t = fun x ⟶ x
```

```
val f: S1.t ⟶ S2.t
```

What flavor for your functor ?

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Stateless, *pure* modules, weak abstraction barrier ⟶ same applications produce equal abstract types

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⟶ Essential for modular implicits (modular type-classes)

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Stateful, effectful modules, strong abstraction barrier ⟶ each application produces new abstract types

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```
module Window (T: Topbar) () = (... : sig type t ... end)
module W1 = Window(T)()
module W2 = Window(T)()
```

```
let f: W1.t ⟶ W2.t = fun x ⟶ x
```

```
W1.t is not equal to W2.t
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module Set(E: Comparable) = (... : sig type t ... end)
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Module expressions – `M`

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- Records of values and types — `struct ... end`
- Developer-side abstraction : ascription — `(M : S)`
- Client-side abstraction : functors — `functor (Y:S) → M`, `functor () → M`
- A small calculus (projections, applications, recursion, etc.) — `M.X`, `F(X)`, `...`

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- Abstract types — `type t`

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→ ML is two languages in one: modules and core

→ modules: explicit, rich types ↔ core: implicit, simpler types

→ simple type-erasing semantics (records and functions)

A rich history of ML-modules

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The beginnings (early 90's)

Early SML versions, existential types, strong sums

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→ Explore the design space of type-systems for ML modules, with three goal: *explainability*, *usability* (error-messages, performance, etc.) and *retrofit*

A perfectible system

Robustness

Semantically equivalent transformations should not break type-checking

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Typing is not stable under β-reduction

```
1 module X = struct type t end
2 module M = F(X)
```

```
module M : sig ... end ✓
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1 (* replaced X by its definition *)
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```

```
The parameter cannot be
eliminated in the result type.
```

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The parameter cannot be
eliminated in the result type.
```

Typing is not stable under β-expansion

```
1 let f : G(F(X)).t → G(F(X)).t
2   = fun x → x
```

```
val f : G(F(X)).t → G(F(X)).t
```

A perfectible system

Robustness

Semantically equivalent transformations should not break type-checking

Typing is not stable under β-reduction

```
1 module X = struct type t end
2 module M = F(X)
```

```
module M : sig ... end ✓
```

```
1 (* replaced X by its definition *)
2 module M = F(struct type t end)
```

```
The parameter cannot be
eliminated in the result type.
```

Typing is not stable under β-expansion

```
1 let f : G(F(X)).t → G(F(X)).t
2   = fun x → x
```

```
val f : G(F(X)).t → G(F(X)).t
```

```
1 module FX = F(X)
2 let f : G(F(X)).t → G(FX).t
3 = fun x → x
```

```
G(F(X)).t and G(FX).t are not equal
```

A perfectible system

Robustness

Semantically equivalent transformations should not break type-checking

Typing is not stable under β-reduction → Signature avoidance

```
module X = struct type t end
module M = F(X)
```

```
module M : sig ... end ✓
```

```
(* replaced X by its definition *)
module M = F(struct type t end)
```

```
The parameter cannot be
eliminated in the result type.
```

Typing is not stable under β-expansion → Module identities

```
let f : G(F(X)).t → G(F(X)).t
  = fun x → x
```

```
val f : G(F(X)).t → G(F(X)).t
```

```
module FX = F(X)
let f : G(F(X)).t → G(FX).t
= fun x → x
```

```
G(F(X)).t and G(FX).t are not equal
```

The signature avoidance problem

abstract type fields + signatures dependencies + fields getting out of scope

The signature avoidance problem

abstract type fields + signatures dependencies + fields getting out of scope

Type-checking fail (visible error)

```
module N = struct
  type t = A of int | B of bool
  module X = struct let x = (A 1) end
end
module M = N.X
```

```
module M : sig val x : N.t end
```

The signature avoidance problem

abstract type fields + signatures dependencies + fields getting out of scope

Type-checking fail (visible error)

```
1 module N = struct
2   type t = A of int | B of bool
3   module X = struct let x = (A 1) end
4 end
5 module M = N.X
```

```
module M : sig val x : N.t end
```

```
1 (* replaced N by its definition *)
2 module M = (struct
3   type t = A of int | B of bool
4   module X = struct let x = (A 1) end
5 end ).X
```

```
The value x has no valid type
if t is hidden.
```

The signature avoidance problem

abstract type fields + signatures dependencies + fields getting out of scope

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Over-abstraction (silent error!)

```
module N = struct
  type t
  module X = struct type u = t
                    type v = t end
end
module M = N.X
```

```
module M : sig type u = N.t
               type v = N.t end
```

The signature avoidance problem

abstract type fields + signatures dependencies + fields getting out of scope

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module M : sig val x : N.t end
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Over-abstraction (silent error!)

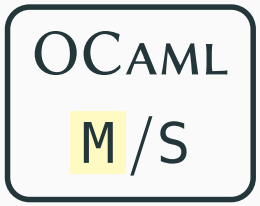
```
1 module N = struct
2   type t
3   module X = struct type u = t
4                     type v = t end
5 end
6 module M = N.X
```

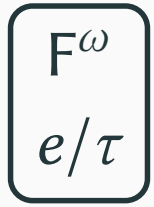
```
module M : sig type u = N.t
               type v = N.t end
```

```
1 (* replaced N by its definition *)
2 module M = (struct
3   type t
4   module X = struct type u = t
5                     type v = t  end
6 end).X
```

```
module M : sig type u
               type v  end
```

The big picture





The big picture

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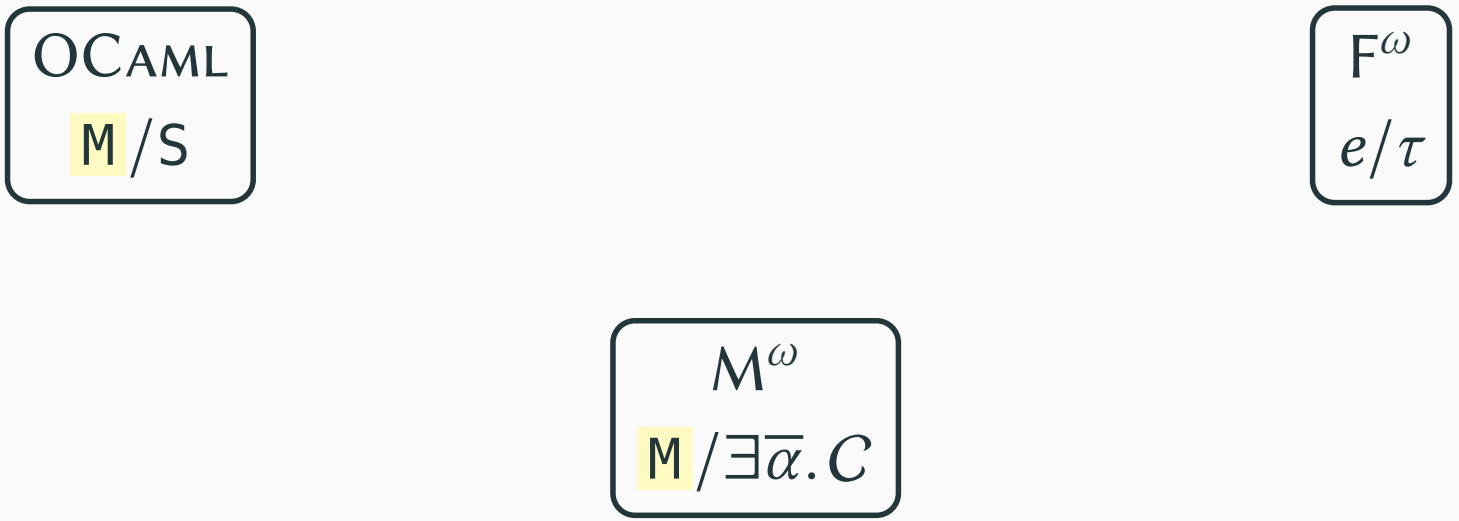

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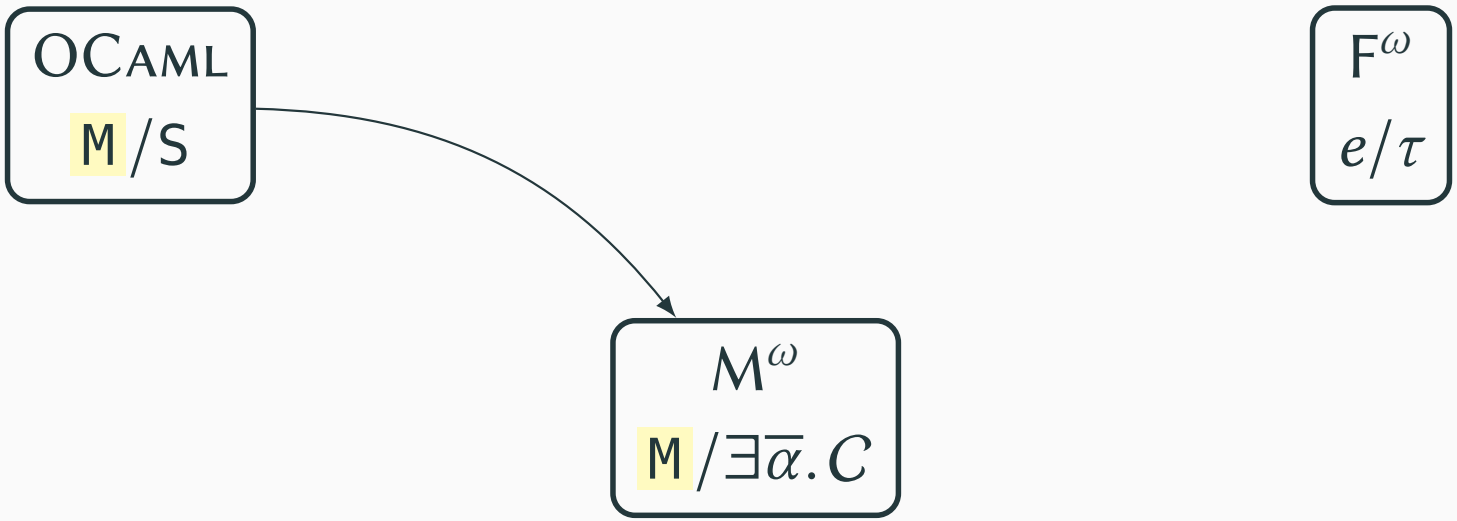

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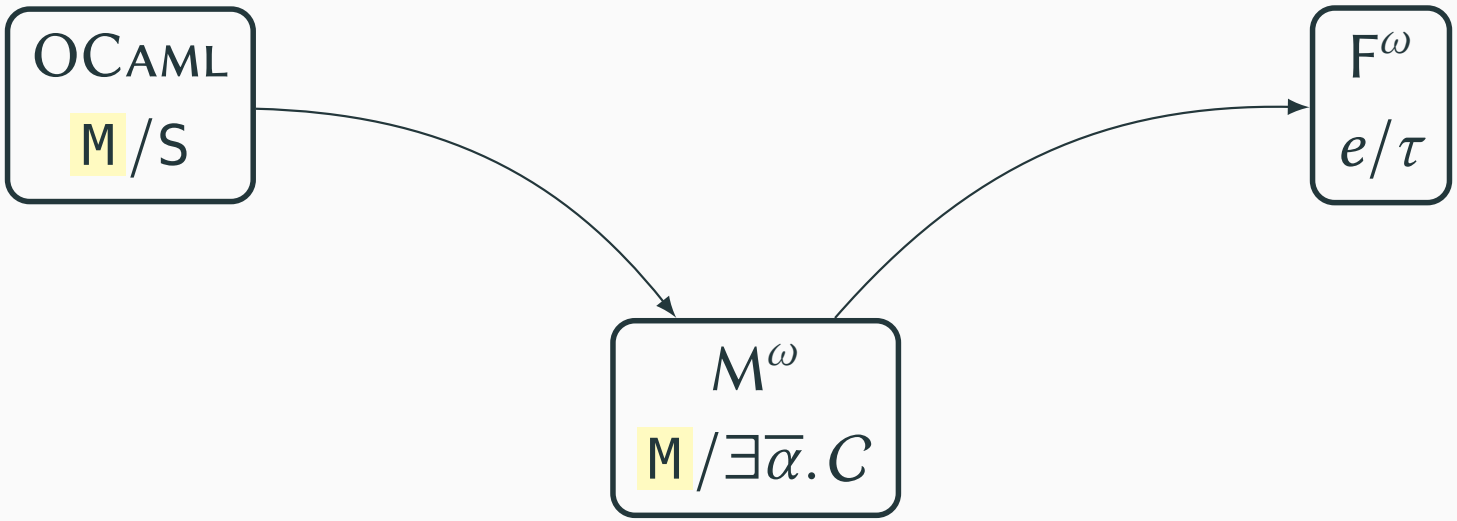

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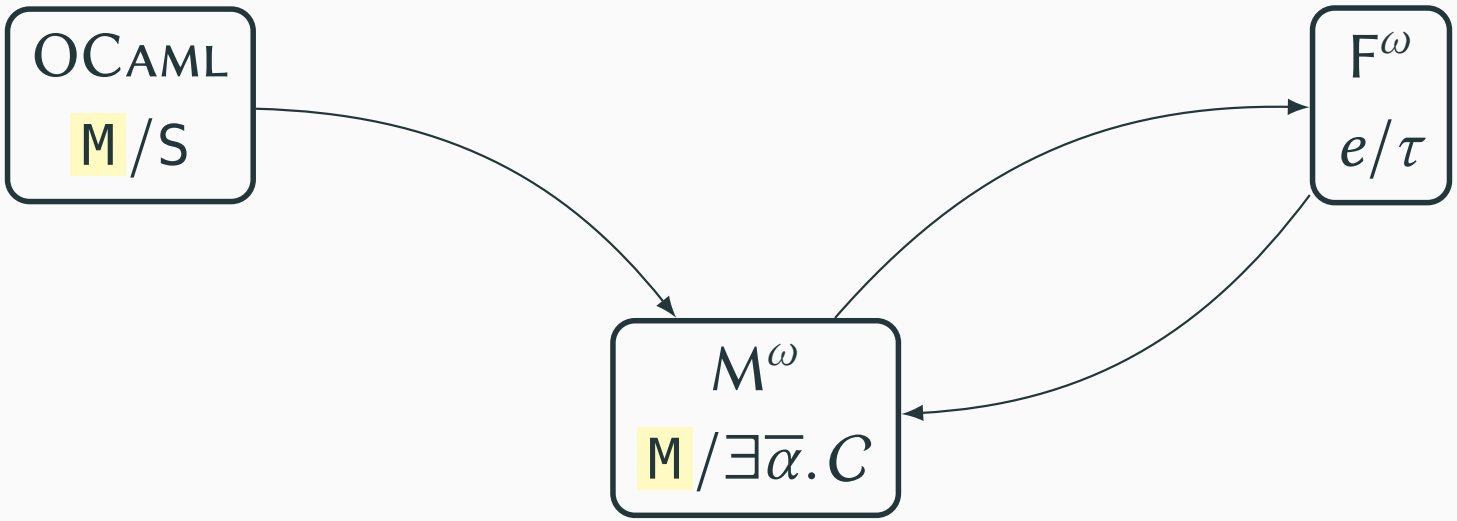

Abstract types have existential type...

Source signature (reference)

- module M
  - module X
    - type t
    - val $x : t$
    - type $u = \text{int} \times t$
  - module F $(Y : \text{sig type } t \text{ end})$
    - type t
    - type $u = Y.t$
  - type $t_1 = F(X).t$
  - type $t_2 = F(X).t$

M^ω signature

- module M
  - module X
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    - type t
    - type $u = Y.t$
  - type $t_1 = F(X).t$
  - type $t_2 = F(X).t$

M^ω signature

- module M
  - module X
  - $\exists\alpha$
    - type $t = \alpha$
    - val $x : t$
    - type $u = \text{int} \times t$
  - module F $(Y : \text{sig type } t \text{ end})$
    - type t
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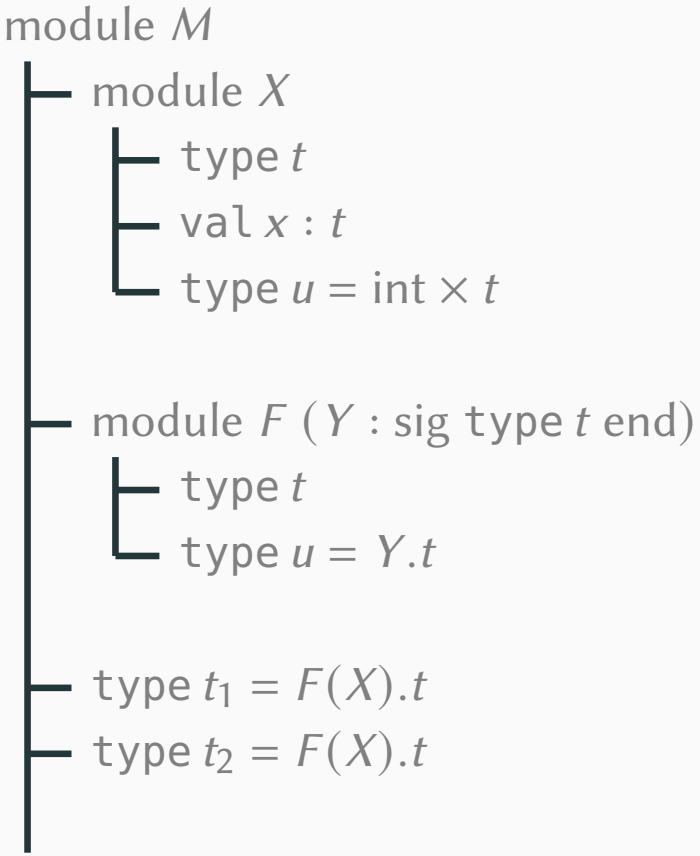
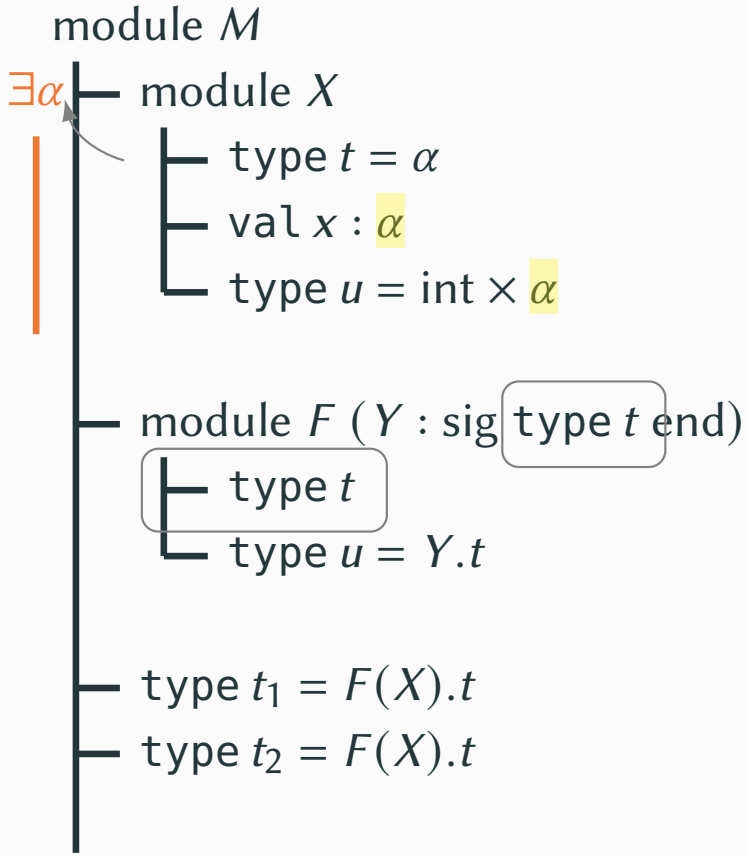


M^{ω} signature



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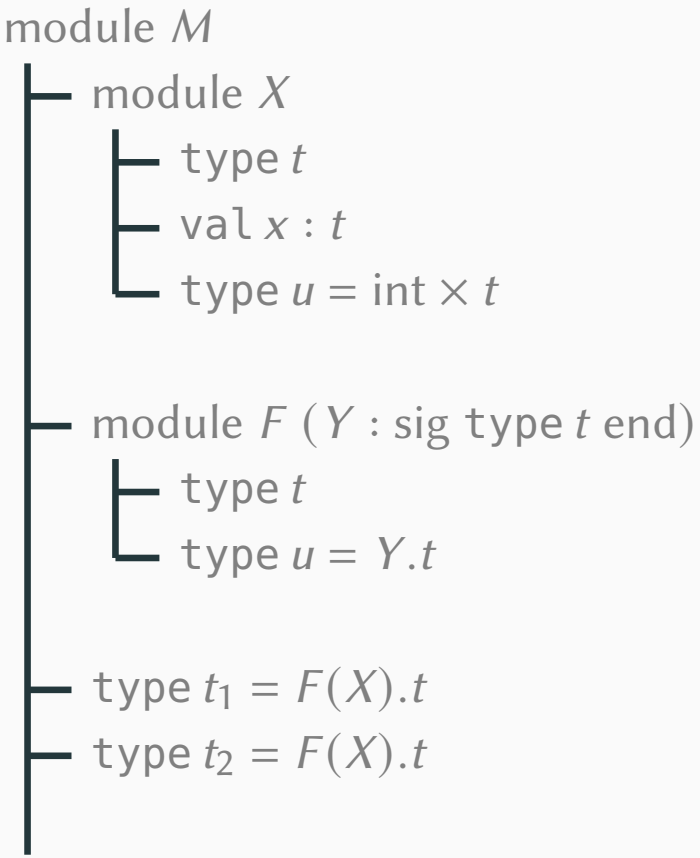
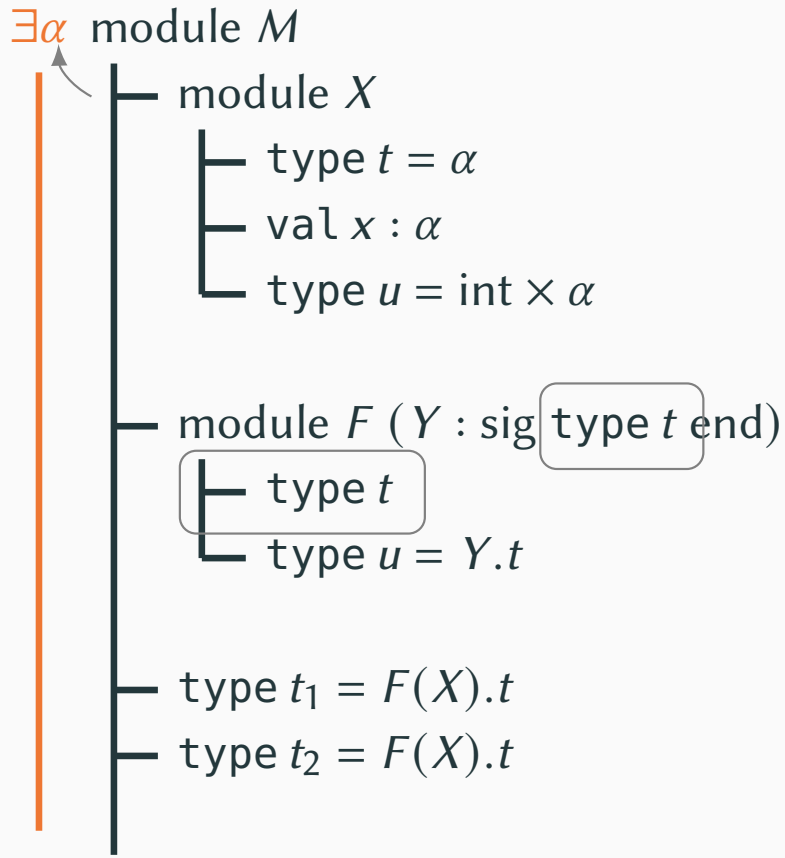


M^{ω} signature



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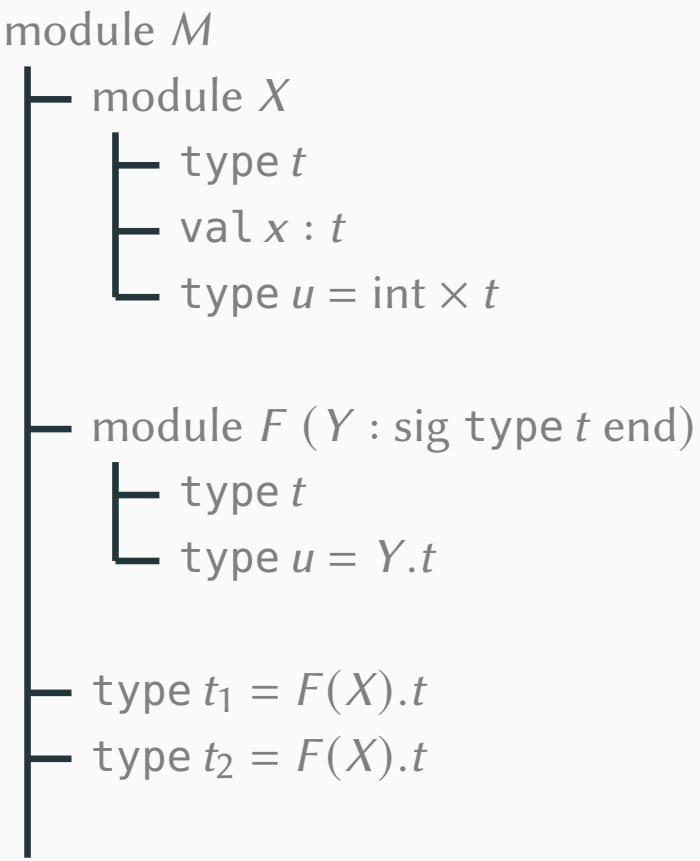
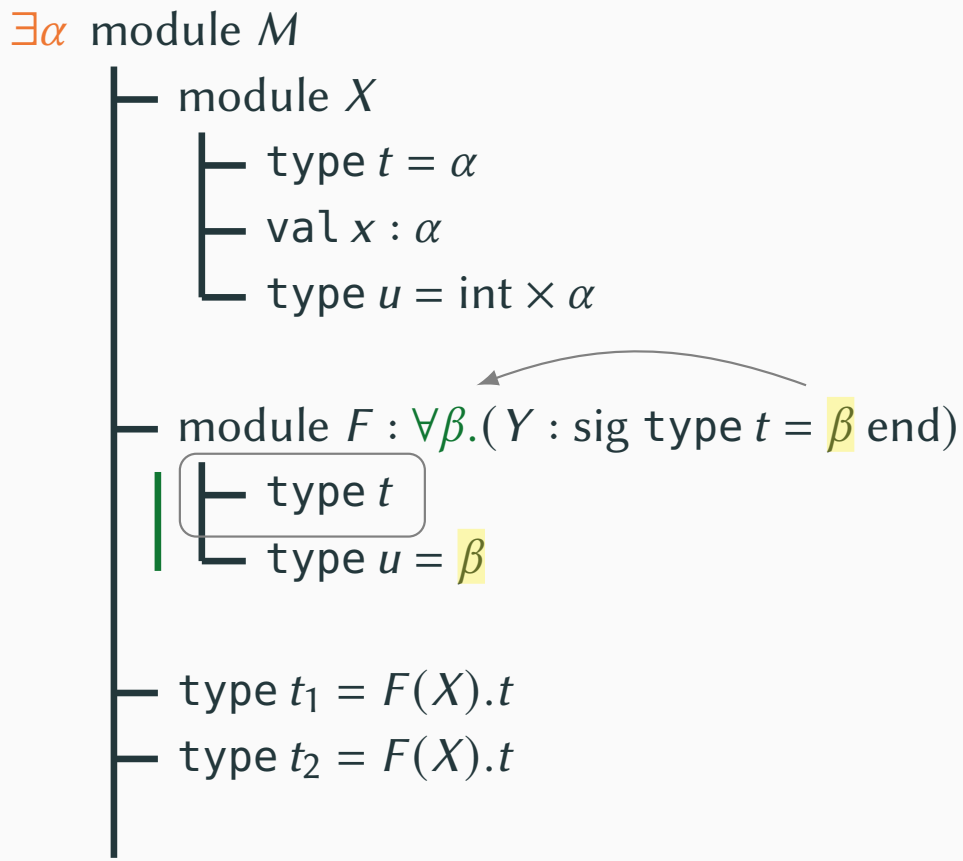


M^{ω} signature



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M^ω signature

- $\exists\alpha$ module M
  - module X
    - type $t = \alpha$
    - val $x : \alpha$
    - type $u = \text{int} \times \alpha$
  - module $F : \forall\beta.(Y : \text{sig type } t = \beta \text{ end})$
    - $\exists\gamma$ type $t = \gamma$
    - type $u = \beta$
  - type $t_1 = F(X).t$
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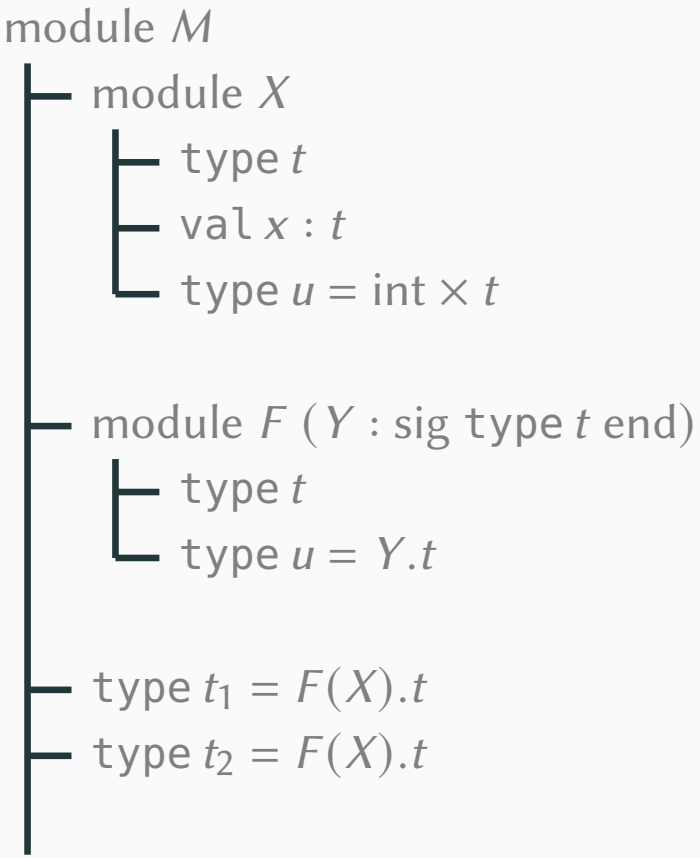
Abstract types have existential type...

Source signature (reference)



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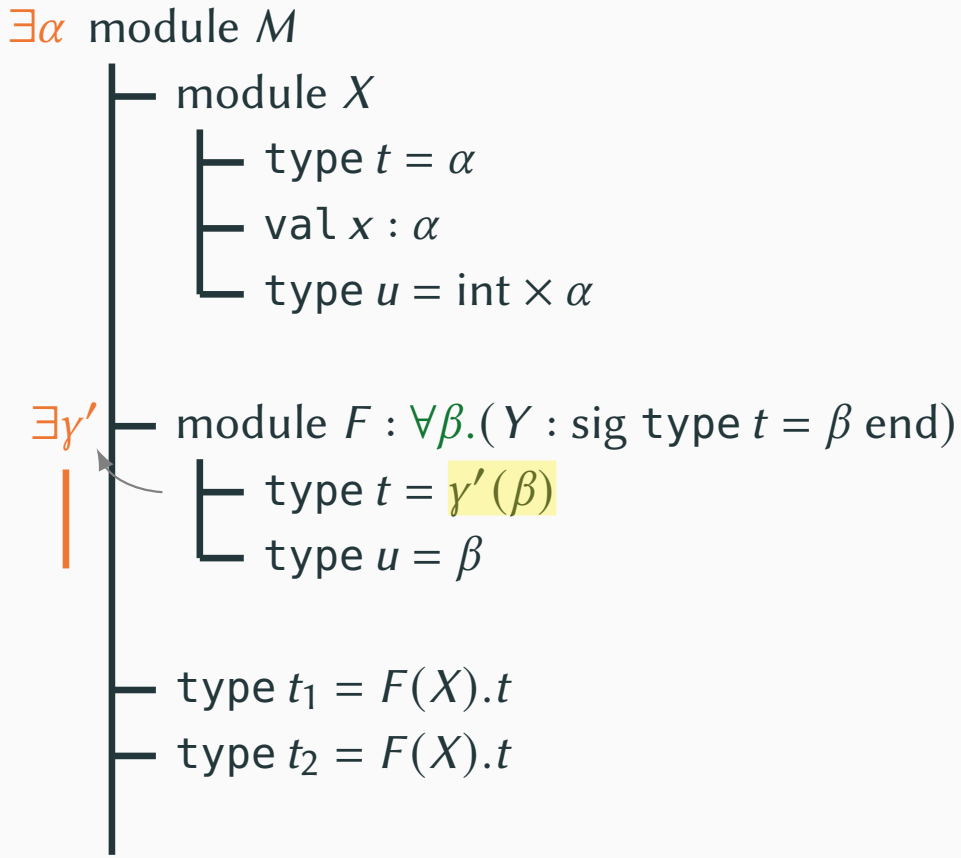

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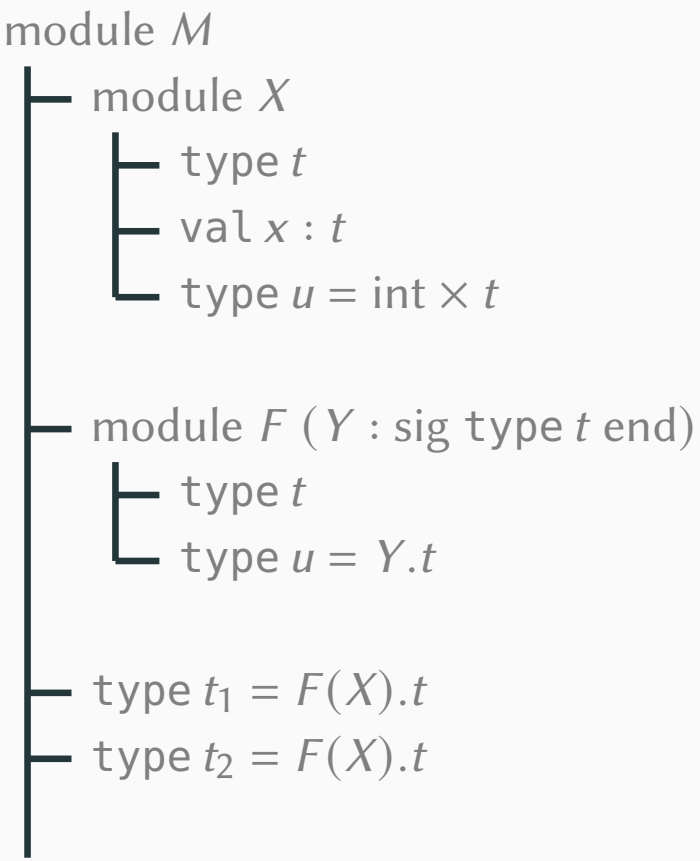
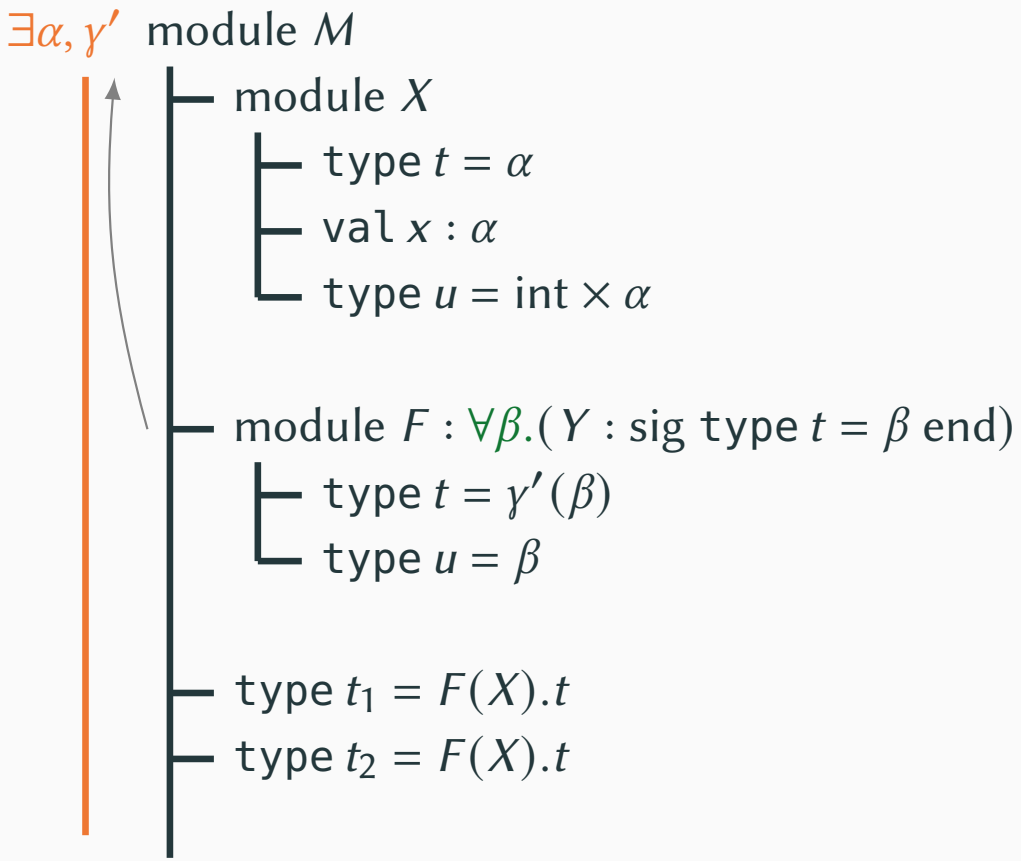


M^ω signature



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    - type $u = Y.t$
  - type $t_1 = F(X).t$
  - type $t_2 = F(X).t$

M^ω signature

$\exists \alpha, \gamma'$ module M

- module X
  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $F : \forall \beta.(Y : \text{sig type } t = \beta \text{ end})$
  - type $t = \gamma'(\beta)$
  - type $u = \beta$
- type $t_1 = \gamma'(\alpha)$
- type $t_2 = F(X).t$

Abstract types have existential type...

Source signature (reference)

- module M
  - module X
    - type t
    - val $x : t$
    - type $u = \text{int} \times t$
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  - type $t = \alpha$
  - val $x : \alpha$
  - type $u = \text{int} \times \alpha$
- module $F : \forall \beta.(Y : \text{sig type } t = \beta \text{ end})$
  - type $t = \gamma'(\beta)$
  - type $u = \beta$
- type $t_1 = \gamma'(\alpha)$
- type $t_2 = \gamma'(\alpha)$

F^ω overview / M^ω overview

F^ω - with primitive records and existential types

$$\tau := \alpha \mid \tau \to \tau \mid \{\overline{\ell : \tau}\} \mid \forall\alpha.\,\tau \mid \exists\alpha.\tau \mid \lambda\alpha.\,\tau \mid () \qquad \text{(types)}$$

$$e := x \mid \lambda(x : \tau).e \mid e\,e \mid \Lambda\alpha.\,e \mid e\,\tau \mid e \mathbin{@} e \mid \{\overline{\ell = e}\} \mid e.\ell$$

$$\mid \text{pack}\,\langle\tau, e\rangle\,\text{as}\,\exists\alpha.\tau \mid \text{unpack}\,\langle\alpha, x\rangle = e\,\text{in}\,e \mid () \qquad \text{(terms)}$$

F^ω overview / M^ω overview

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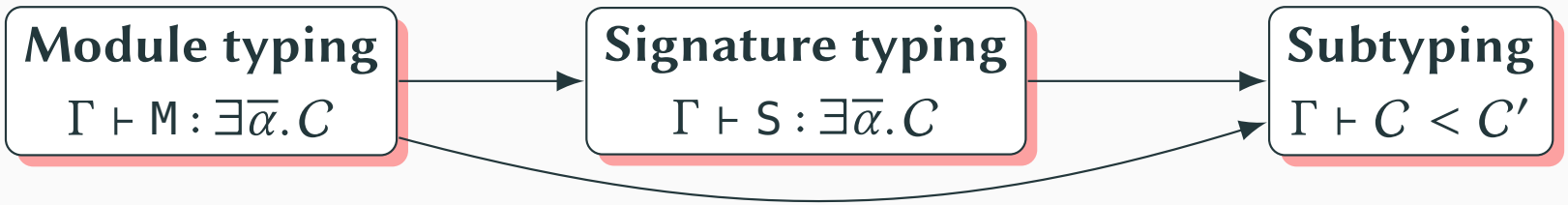
Momega



F^ω overview / M^ω overview

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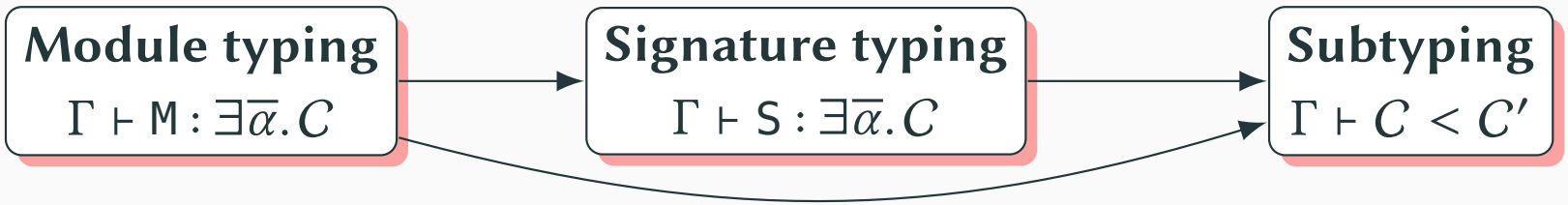

$\to$ M^ω signatures $\exists\overline{\alpha}.C$ are just syntactic sugar for F^ω types

F^ω overview / M^ω overview

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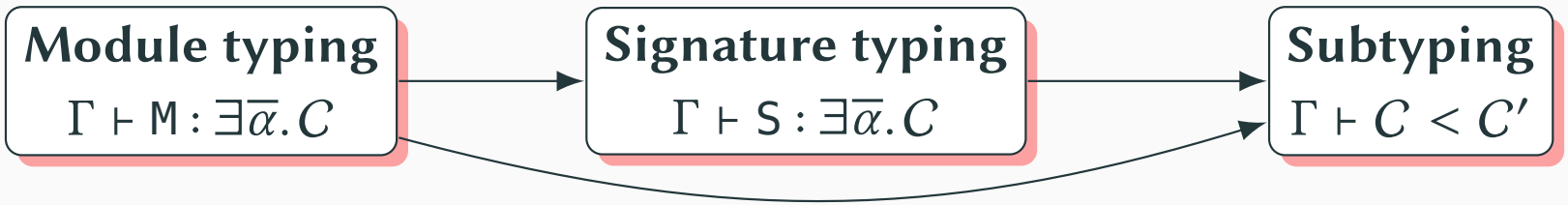
Momega



→ M^ω signatures $\exists\overline{\alpha}.C$ are just syntactic sugar for F^ω types

→ Soundness is shown by building a well-typed *evidence term* in F^ω

$$\Gamma \vdash \mathsf{M} : \exists\overline{\alpha}.C \rightsquigarrow e \qquad\qquad \Gamma \vdash C <: C' \rightsquigarrow f$$

Example of encoding into F^{ω}

Source code

```
module type S =
  sig type t val x : t end
```

```
module M = struct
  module X1 = (struct
    type t = int
    let x = 42
  end : S )
  module X2 = (struct
    type t = X1.t * bool
    let x = (X1.x, false)
  end : S )
end
```

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M^{ω} Signature

$\exists \alpha, \beta.$ **module** M : **sig**
  module X1 : **sig**
    type t = α
    val x : α
  end
  module X2 : **sig**
    type t = β
    val x : β
  end
end

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end
```

M^{ω} Signature

```
∃α, β. module M : sig
  module X1 : sig
    type t = α
    val x : α
  end
  module X2 : sig
    type t = β
    val x : β
  end
end
```

F^{ω} type

$$\exists \alpha, \beta. \left\{ \begin{array}{l} \ell_{X_1} : \left\{ \begin{array}{l} \ell_t : \langle\langle \alpha \rangle\rangle \\ \ell_x : \alpha \end{array} \right\} \\ \ell_{X_2} : \left\{ \begin{array}{l} \ell_t : \langle\langle \beta \rangle\rangle \\ \ell_x : \beta \end{array} \right\} \end{array} \right\}$$

where

$$\langle\langle \tau \rangle\rangle \triangleq \forall\, \varphi\, .\, \varphi\, \tau \to \varphi\, \tau$$

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M^{ω} Signature

```
∃α, β. module M : sig
  module X1 : sig
    type t = α
    val x : α
  end
  module X2 : sig
    type t = β
    val x : β
  end
end
```

F^{ω} type

$$\exists \alpha, \beta. \left\{ \begin{array}{l} \ell_{X_1} : \left\{ \begin{array}{l} \ell_t : \langle\langle \alpha \rangle\rangle \\ \ell_x : \alpha \end{array} \right\} \\ \ell_{X_2} : \left\{ \begin{array}{l} \ell_t : \langle\langle \beta \rangle\rangle \\ \ell_x : \beta \end{array} \right\} \end{array} \right\}$$

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Encoded module

$$e =$$

Example of encoding into F^{ω}

Source code

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end
```

M^{ω} Signature

```
∃α, β. module M : sig
  module X1 : sig
    type t = α
    val x : α
  end
  module X2 : sig
    type t = β
    val x : β
  end
end
```

F^{ω} type

$$\exists \alpha, \beta. \left\{ \begin{array}{l} \ell_{X_1} : \left\{ \begin{array}{l} \ell_t : \langle\langle \alpha \rangle\rangle \\ \ell_x : \alpha \end{array} \right\} \\ \ell_{X_2} : \left\{ \begin{array}{l} \ell_t : \langle\langle \beta \rangle\rangle \\ \ell_x : \beta \end{array} \right\} \end{array} \right\}$$

where

$$\langle\langle \tau \rangle\rangle \triangleq \forall\, \varphi\, .\, \varphi\, \tau \to \varphi\, \tau$$

Encoded module

$e =$ $\{\ell_t = \langle\langle \mathsf{int} \rangle\rangle, \ell_x = 42\}$

Example of encoding into F^{ω}

Source code

```
module type S =
  sig type t val x : t end
```

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module M = struct
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  end : S )
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M^{ω} Signature

```
∃α, β. module M : sig
  module X1 : sig
    type t = α
    val x : α
  end
  module X2 : sig
    type t = β
    val x : β
  end
end
```

F^{ω} type

$$\exists \alpha, \beta. \left\{ \begin{array}{l} \ell_{X_1} : \left\{ \begin{array}{l} \ell_t : \langle\langle \alpha \rangle\rangle \\ \ell_x : \alpha \end{array} \right\} \\ \ell_{X_2} : \left\{ \begin{array}{l} \ell_t : \langle\langle \beta \rangle\rangle \\ \ell_x : \beta \end{array} \right\} \end{array} \right\}$$

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    type t = α
    val x : α
  end
  module X2 : **sig**
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    val x : β
  end
end

F^{ω} type

$$\exists \alpha, \beta. \left\{ \begin{array}{l} \ell_{X_1} : \left\{ \begin{array}{l} \ell_t : \langle\langle \alpha \rangle\rangle \\ \ell_x : \alpha \end{array} \right\} \\ \ell_{X_2} : \left\{ \begin{array}{l} \ell_t : \langle\langle \beta \rangle\rangle \\ \ell_x : \beta \end{array} \right\} \end{array} \right\}$$

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$$e = \mathsf{unpack}\langle \alpha, y_1 \rangle = \mathsf{pack}\langle \mathtt{int}, \{\ell_t = \langle\langle \mathtt{int} \rangle\rangle, \ell_x = 42\} \rangle \text{ in } \mathsf{pack}\langle \alpha \times \mathtt{bool}, \{\ell_t = \langle\langle \alpha \times \mathtt{bool} \rangle\rangle, \ell_x = (y_1.x, \bot)\} \rangle$$

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Extrusion and skolemization

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$$\vdash \lambda b.\ \text{if } b \quad \text{then } (\text{pack } \langle \text{int}, 42 \rangle \text{ as } \exists\alpha.\alpha)$$
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  $(\exists\alpha.\tau, \sigma) \rightsquigarrow \exists\alpha.(\tau, \sigma)$ ✓ (*repacking* pattern)
- Through arrow types (extend scope outside of applicative functors):
  $(\tau \to \exists\alpha.\sigma) \rightsquigarrow \exists\alpha.(\tau \to \sigma)$ × unsound in general !

$$\vdash \lambda b.\ \text{if } b \ \text{ then } (\text{pack } \langle \text{int}, 42\rangle \text{ as } \exists\alpha.\alpha) \qquad : \text{bool} \to \exists\alpha.\alpha$$
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Some existentials are only used for abstraction, not for *heterogeneity*

F^{ω} with transparent existentials - Adding new constructs

$$\tau := \ \ldots \mid \exists^{\blacktriangledown} \alpha . \tau$$

$$\begin{aligned} e := \ & \ldots \mid \text{pack} \langle \tau, e \rangle \text{ as } \exists^{\blacktriangledown} \alpha . \tau \\ & \mid \text{unpack}^{\blacktriangledown} \langle \alpha, x \rangle = e \text{ in } e \end{aligned}$$

F^{ω} with transparent existentials - Adding new constructs

$$
\begin{aligned}
\tau &:= \ \ldots \mid \exists^{\blacktriangledown} \alpha.\, \tau \\
&\quad\ \mid \exists^{\triangledown \tau} \alpha.\, \tau \\
e &:= \ \ldots \mid \text{pack } \langle \tau, e \rangle \text{ as } \exists^{\blacktriangledown} \alpha.\, \tau \\
&\quad\ \mid \text{unpack}^{\blacktriangledown}\ \langle \alpha, x \rangle = e \text{ in } e
\end{aligned}
$$

F^{ω} with transparent existentials - Adding new constructs

$$
\begin{aligned}
\tau ::= \ & \ldots \mid \exists^{\blacktriangledown} \alpha.\, \tau \\
& \mid \exists^{\nabla \tau} \alpha.\, \tau \\
e ::= \ & \ldots \mid \mathsf{pack}\ \langle \tau, e \rangle\ \mathsf{as}\ \exists^{\blacktriangledown} \alpha.\, \tau \\
& \mid \mathsf{unpack}^{\blacktriangledown}\ \langle \alpha, x \rangle = e\ \mathsf{in}\ e \\
& \mid \mathsf{pack}\ e\ \mathsf{as}\ \exists^{\nabla \tau} \alpha.\, \sigma \\
& \mid \mathsf{repack}^{\nabla}\ \langle \alpha, x \rangle = e_1\ \mathsf{in}\ e_2 \\
& \mid \mathsf{seal}\ e \\
& \mid \mathsf{lift}^{\rightarrow} e \\
& \mid \mathsf{lift}^{\forall} e
\end{aligned}
$$

F^{ω} with transparent existentials - Typing rules

F-LiftArr

$$\frac{\Gamma \vdash e : \sigma_1 \to (\exists^{\triangledown\tau} \alpha.\, \sigma_2)}{\Gamma \vdash \mathsf{lift}^{\to} e : \exists^{\triangledown\tau} \alpha.\, (\sigma_1 \to \sigma_2)}$$

F^{ω} with transparent existentials - Typing rules

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F-Repack

$$\frac{\Gamma \vdash e_1 : \exists^{\nabla\tau} \alpha.\, \sigma_1 \qquad \Gamma, \alpha, x : \sigma_1 \vdash e_2 : \sigma_2}{\Gamma \vdash \mathsf{repack}^{\nabla} \langle \alpha, x \rangle = e_1 \text{ in } e_2 : \exists^{\nabla\tau} \alpha.\, \sigma_2}$$

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F-Hide

$$\frac{\Gamma \vdash \exists^{\nabla\tau} \alpha.\, \sigma : \star \qquad \Gamma \vdash e : \sigma[\alpha \mapsto \tau]}{\Gamma \vdash \mathsf{pack}\ e\ \mathsf{as}\ \exists^{\nabla\tau} \alpha.\, \sigma : \exists^{\nabla\tau} \alpha.\, \sigma}$$

F-Seal

$$\frac{\Gamma \vdash e : \exists^{\nabla\tau} \alpha.\, \sigma}{\Gamma \vdash \mathsf{seal}\ e : \exists^{\blacktriangledown} \alpha.\, \sigma}$$

F-LiftAll

$$\frac{\Gamma \vdash e : \forall \alpha.\, \exists^{\nabla\tau} \beta.\, \sigma}{\Gamma \vdash \mathsf{lift}^{\forall} e : \exists^{\nabla \lambda \alpha.\, \tau} \beta'.\, \forall \alpha.\, \sigma[\beta \mapsto (\beta'\ \alpha)]}$$

F^{ω} with transparent existentials - is actually vanilla F^{ω}!

$$
\begin{array}{lcl}
e_0 & \triangleq & \left\{ \begin{array}{lcl}
\mathsf{Pack} & = & \Lambda\alpha.\,\Lambda\varphi.\,\lambda(x : \varphi\,\alpha).\,x \\
\mathsf{Seal} & = & \Lambda\alpha.\,\Lambda\varphi.\,\lambda(x : \varphi\,\alpha).\,\mathsf{pack}\,\langle\alpha, x\rangle\ \mathsf{as}\ \exists^{\blacktriangledown}\alpha.\,\varphi\,\alpha \\
\mathsf{Repack} & = & \Lambda\alpha.\,\Lambda\varphi.\,\lambda(x : \mathbb{E}\,\alpha\,\varphi).\,\Lambda\psi.\,\lambda(f : \forall\alpha.\,\varphi\,\alpha \to \psi\,\alpha).\,(f\,\alpha\,x) \\
\mathsf{Lift}^{\to} & = & \Lambda\alpha.\,\Lambda\varphi.\,\Lambda\beta.\,\lambda(f : \beta \to \varphi\,\alpha).\,f \\
\mathsf{Lift}^{\forall} & = & \Lambda\alpha.\,\Lambda\varphi.\,\lambda(x : (\forall\beta.\,\varphi\,\beta(\alpha\,\beta))).\,x
\end{array} \right\} \\
\tau_0 & \triangleq & \lambda\alpha.\,\lambda\,\varphi\,.\,\varphi\,\alpha \\
\tau_{\mathbb{E}} & \triangleq & \exists^{\blacktriangledown}\mathbb{E}\,.\left\{ \begin{array}{lcl}
\mathsf{Pack} & : & \forall\alpha.\,\forall\,\varphi\,.\,\varphi\,\alpha \to \mathbb{E}\,\alpha\,\varphi \\
\mathsf{Seal} & : & \forall\alpha.\,\forall\,\varphi\,.\,\mathbb{E}\,\alpha\,\varphi \to \exists^{\blacktriangledown}\alpha.\,\varphi\,\alpha \\
\mathsf{Repack} & : & \forall\alpha.\,\forall\,\varphi\,.\,\mathbb{E}\,\alpha\,\varphi \to \forall\,\psi\,.\,(\forall\alpha.\,\varphi\,\alpha \to \psi\,\alpha) \to \mathbb{E}\,\alpha\,\psi \\
\mathsf{Lift}^{\to} & : & \forall\alpha.\,\forall\,\varphi\,.\,\forall\beta.\,(\beta \to \mathbb{E}\,\alpha\,\varphi) \to \mathbb{E}\,\alpha\,(\lambda\alpha.\,\beta \to \varphi\,\alpha) \\
\mathsf{Lift}^{\forall} & : & \forall\alpha.\,\forall\,\varphi\,.\,(\forall\beta.\mathbb{E}\,(\alpha\,\beta)\,\,(\varphi\,\beta)) \to \mathbb{E}\,\alpha\,\,(\lambda\alpha.\forall\beta.\,\varphi\,\beta\,\,(\alpha\,\beta))
\end{array} \right\} \\
e_{\mathbb{E}} & \triangleq & \mathsf{pack}\,\langle\tau_0, e_0\rangle\ \mathsf{as}\ \tau_{\mathbb{E}}
\end{array}
$$

Extrusion and skolemization (with transparent existentials)

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  $(\exists^{\blacktriangledown}\alpha.\,\tau, \sigma) \rightsquigarrow \exists^{\blacktriangledown}\alpha.\,(\tau, \sigma)$ ✓ (*repacking* pattern)
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  $(\forall\beta.\exists^{\nabla\rho}\alpha.\tau) \rightsquigarrow \exists^{\nabla(\lambda\beta.\rho)}\alpha'.(\forall\beta.\tau[\alpha \mapsto (\alpha'\ \beta)])$ ✓ (using lift$^{\forall}$)

M^{ω}- Properties

Theorem (Completeness)

$\Gamma \vdash \mathtt{M} : \exists^{\diamond}\overline{\alpha}.C$ *implies* $\Gamma \vdash \mathtt{M} : \exists^{\diamond}\overline{\alpha}.C \rightsquigarrow e$ *for some* e

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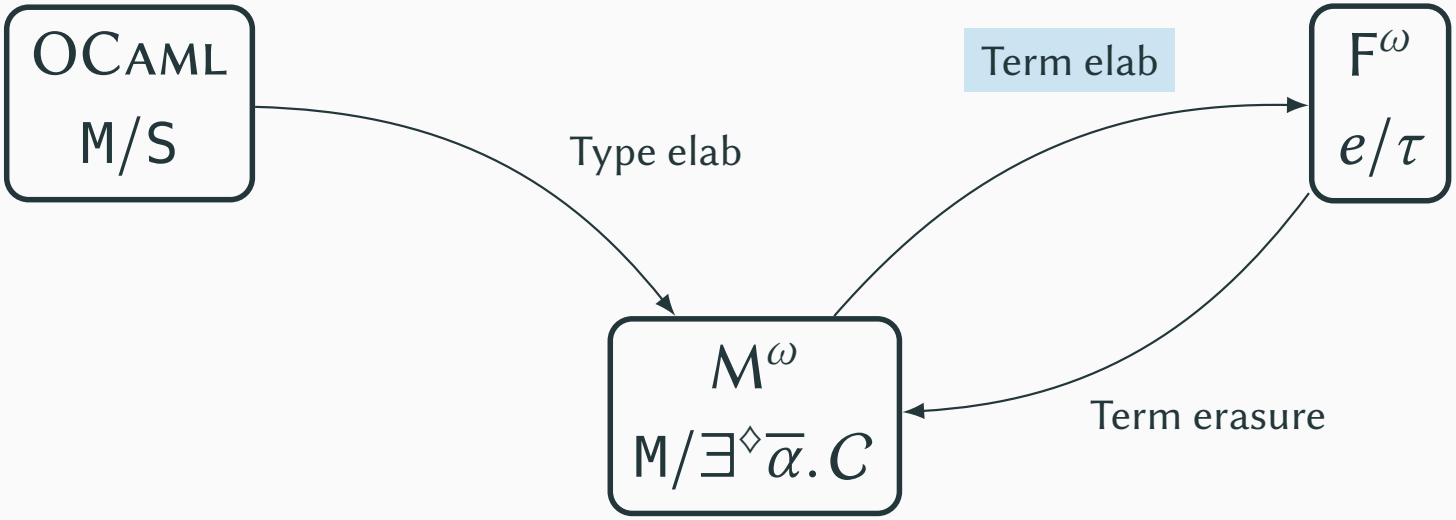
Theorem (Soundness)

$\Gamma \vdash \mathtt{M} : \exists^{\diamond}\overline{\alpha}.C \rightsquigarrow e$ *implies* $\Gamma \vdash e : \exists^{\diamond}\overline{\alpha}.C$ *(in* F^{ω}*)*

(e *and* $\mathtt{M}$ *have the same untyped semantics)*

The big picture (again)



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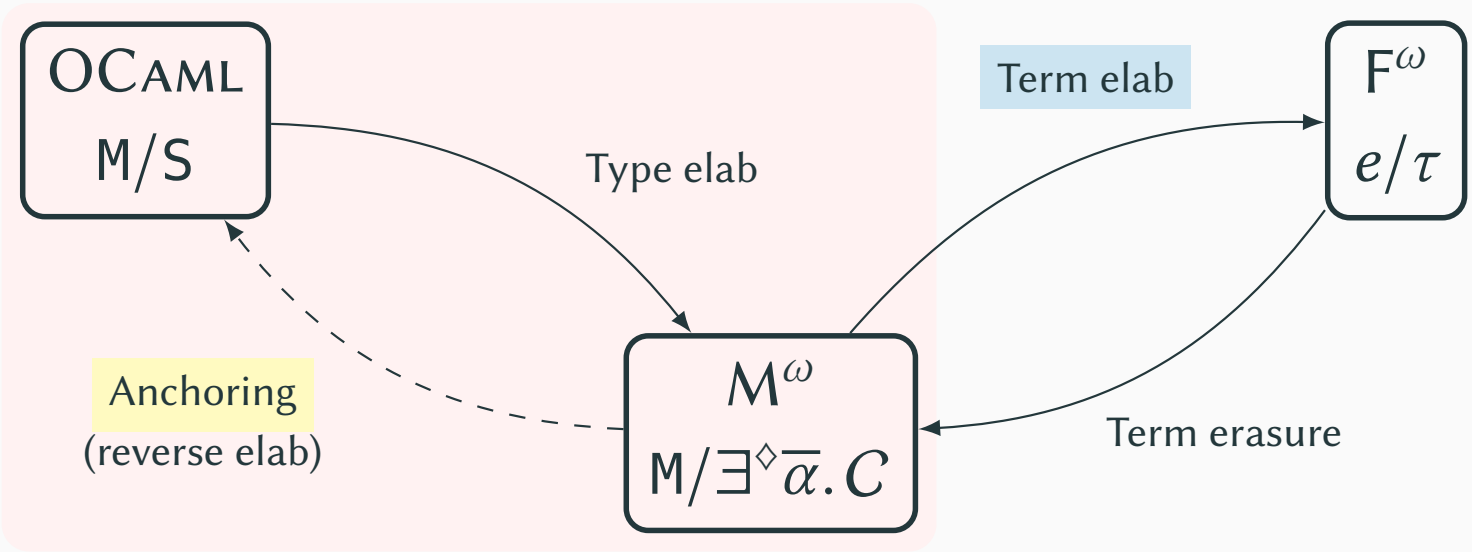

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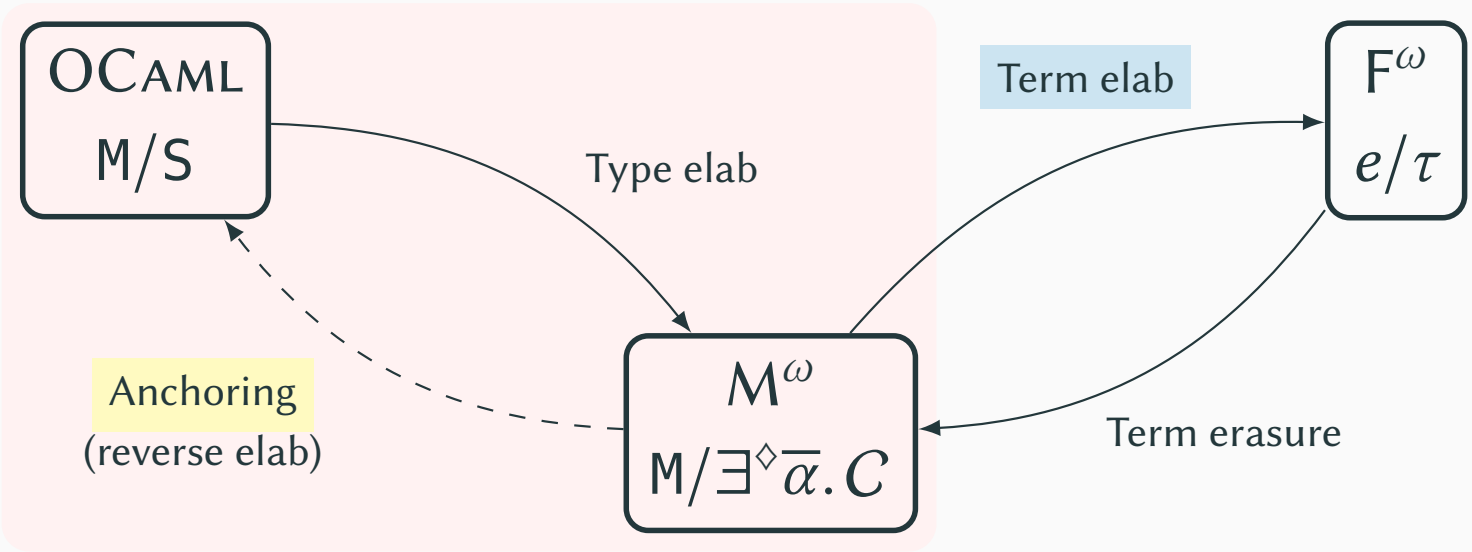

- ✓ Significant subset of OCaml
- ✓ Simple, classic scheme
- ✓ Clear treatment of avoidance
- ✓ Transparent signatures (aliasing), abstract signatures, etc.

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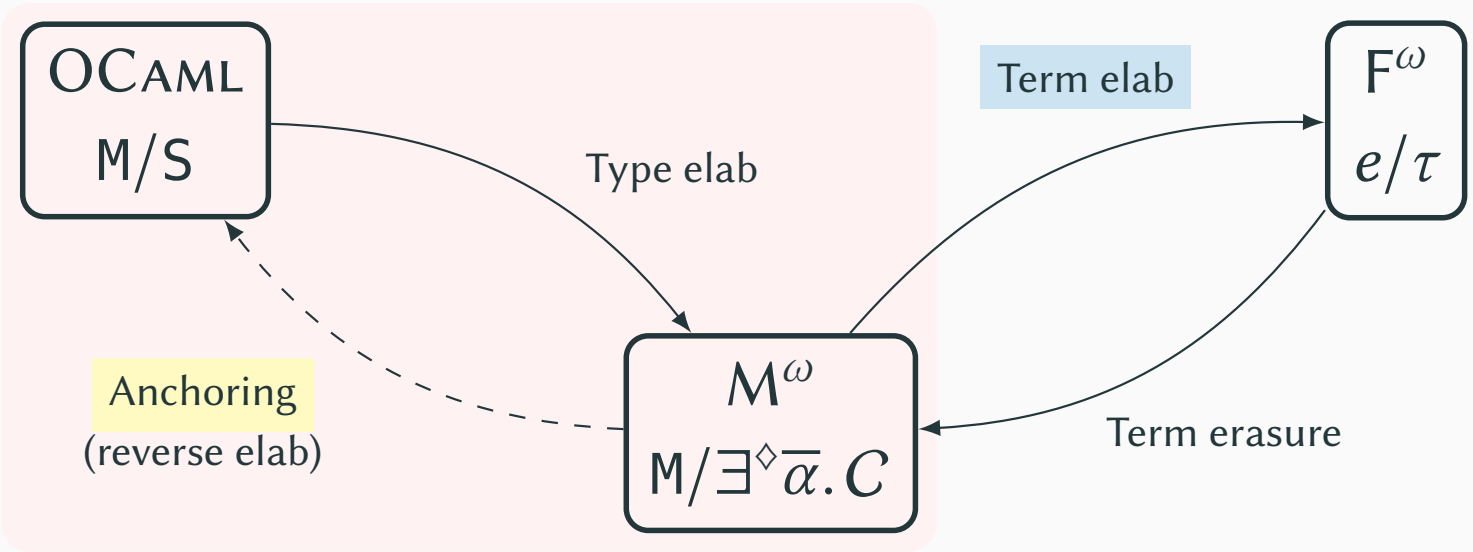

- ✓ Significant subset of OCaml
- ✓ Simple, classic scheme
- ✓ Clear treatment of avoidance
- ✓ Transparent signatures (aliasing), abstract signatures, etc.
- ~ Explainability (elaboration)
- ~ Keeping user-defined names
- × Major change

ZipML

Avoiding signature avoidance

```
(* replaced N by its definition *)
module M = (struct
  type t = A of int | B of bool
  module X = struct let x = (A 1) end
end).X
```

OCaml:

```
The value x has no valid type if t is hidden.
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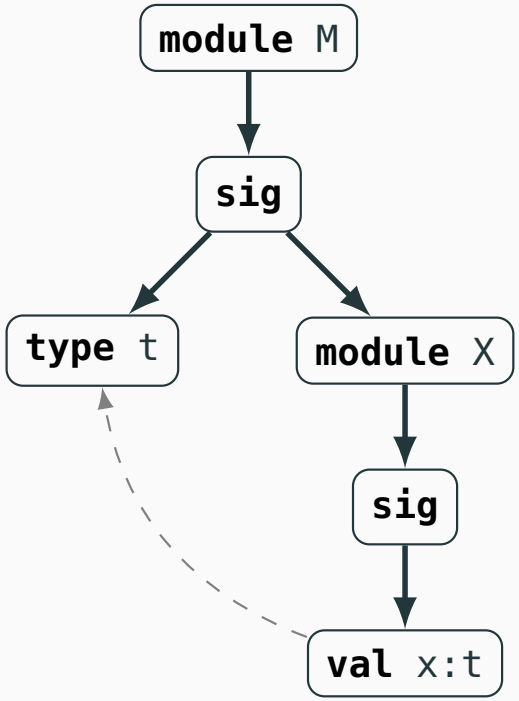

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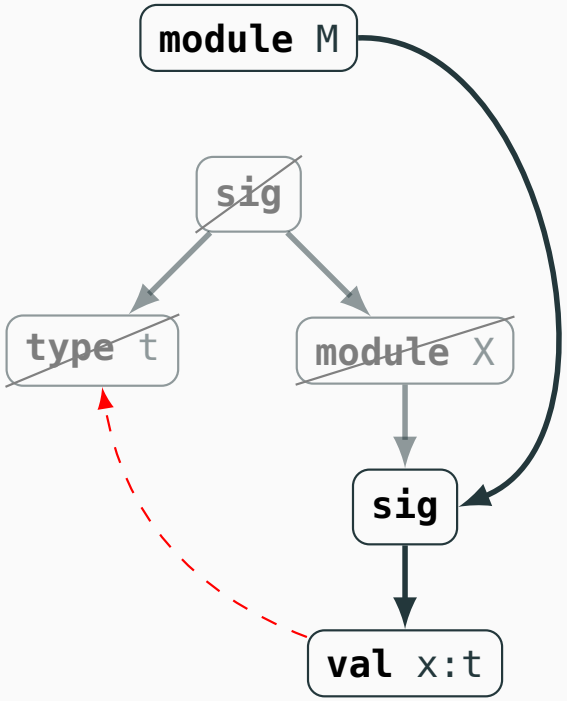

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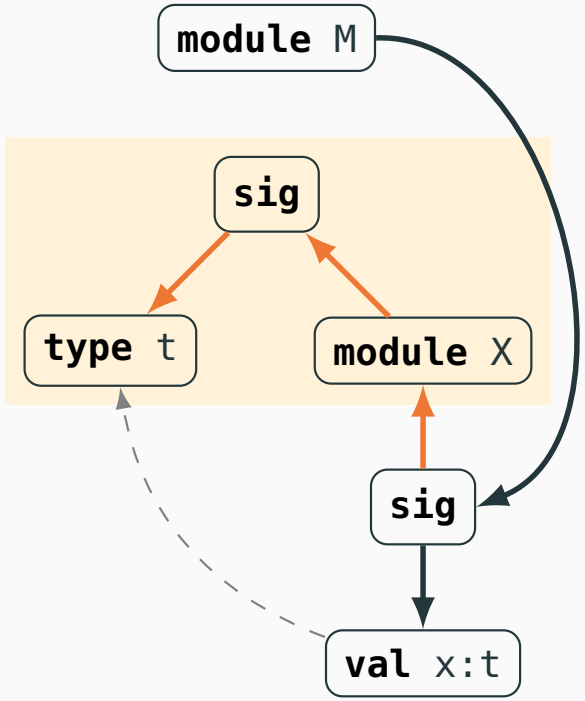

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module M :
< type t = A of int | B of bool>
sig val x : t end
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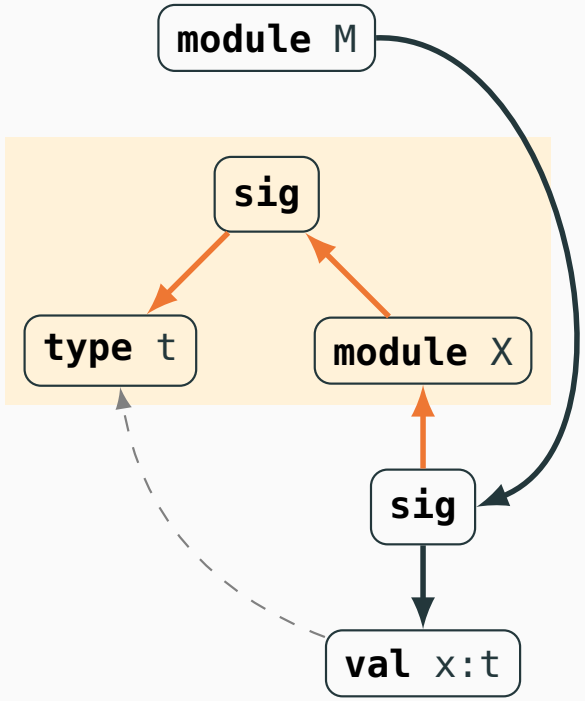

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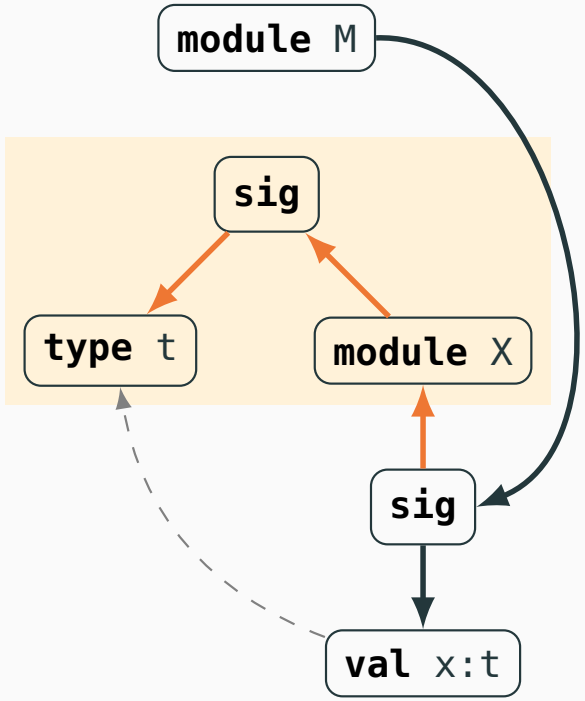

→ Extend the signature language with zippers – $\langle \gamma \rangle$ S

→ The *zipper context* γ acts as an hidden typing environment attached to S

Zipper simplification

```
(* replaced N by its definition *)
module M = (struct
  type t
  type u = t list
  module X = struct type v = t let x : u = [] end
end).X
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<type t >                  sig type v = t val x : t list end
                           sig type v   val x : v list end <type t = v >
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                           sig type v     val x : v list end <type t = v >
                           sig type v     val x : v list end
```

→ valid signature in the source syntax!

Typing with zippers

Zipper introduction

Projections on structure never fails
(no signature avoidance)

$$\textsc{Typ-M-ProjA}\ \frac{\Gamma \vdash \mathtt{M} : \langle \gamma \rangle\ \mathtt{sig}\ \overline{\mathtt{D}}, \mathtt{module}\ X : \mathtt{S}, \overline{\mathtt{D}}'\ \mathtt{end}}{\Gamma \vdash \mathtt{M}.X : \left\langle \gamma \,;\, \overline{\mathtt{D}} \right\rangle \mathtt{S}}$$

Typing with zippers

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Zipper removal (subtyping)

Zippers can disappear through explicit signature ascription (including `.mli`)

Typ-M-Ascr

$$\frac{\Gamma \vdash \mathtt{S} \qquad \Gamma \vdash \mathtt{M} : \langle \gamma \rangle\, \mathtt{S}' \qquad \Gamma, \gamma \vdash \mathtt{S}' \leqslant \mathtt{S}}{\Gamma \vdash^{\diamond} (\mathtt{M} : \mathtt{S}) : \mathtt{S}}$$

Typing with zippers

Zipper introduction

Projections on structure never fails (no signature avoidance)

Typ-M-ProjA

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Zipper removal (subtyping)

Zippers can disappear through explicit signature ascription (including `.mli`)

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Zipper simplification

Garbage-collection of useless fields

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Typing with zippers

Zipper introduction

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Theorem (Preservation of typing)

Simplification never prevents further typing (no over-abstraction)

ZipML- Properties

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Theorem (Soundness)

Well-typed programs do not crash

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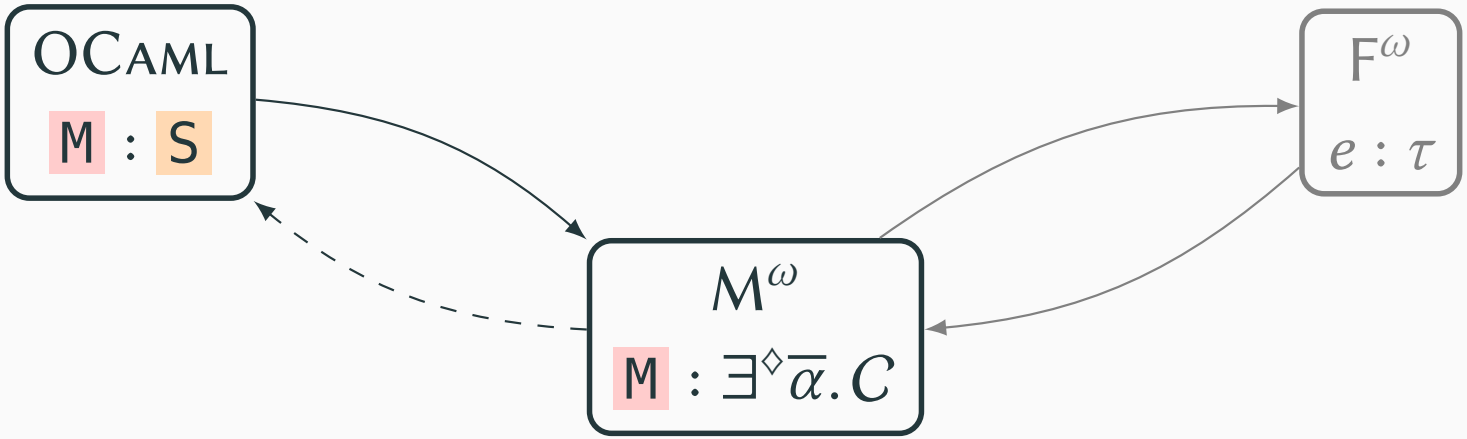
Zippers can be used to represent quantified variables and to simulate extrusion

The big picture (again) (again)



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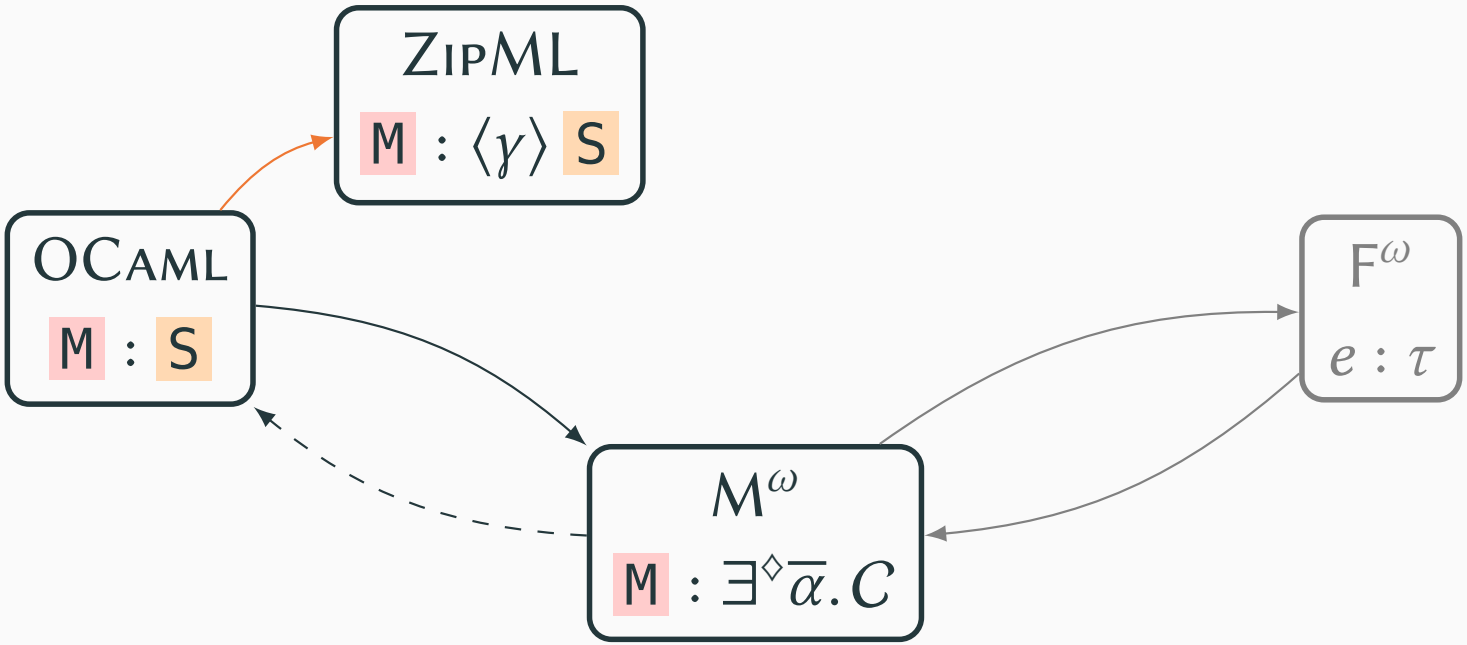

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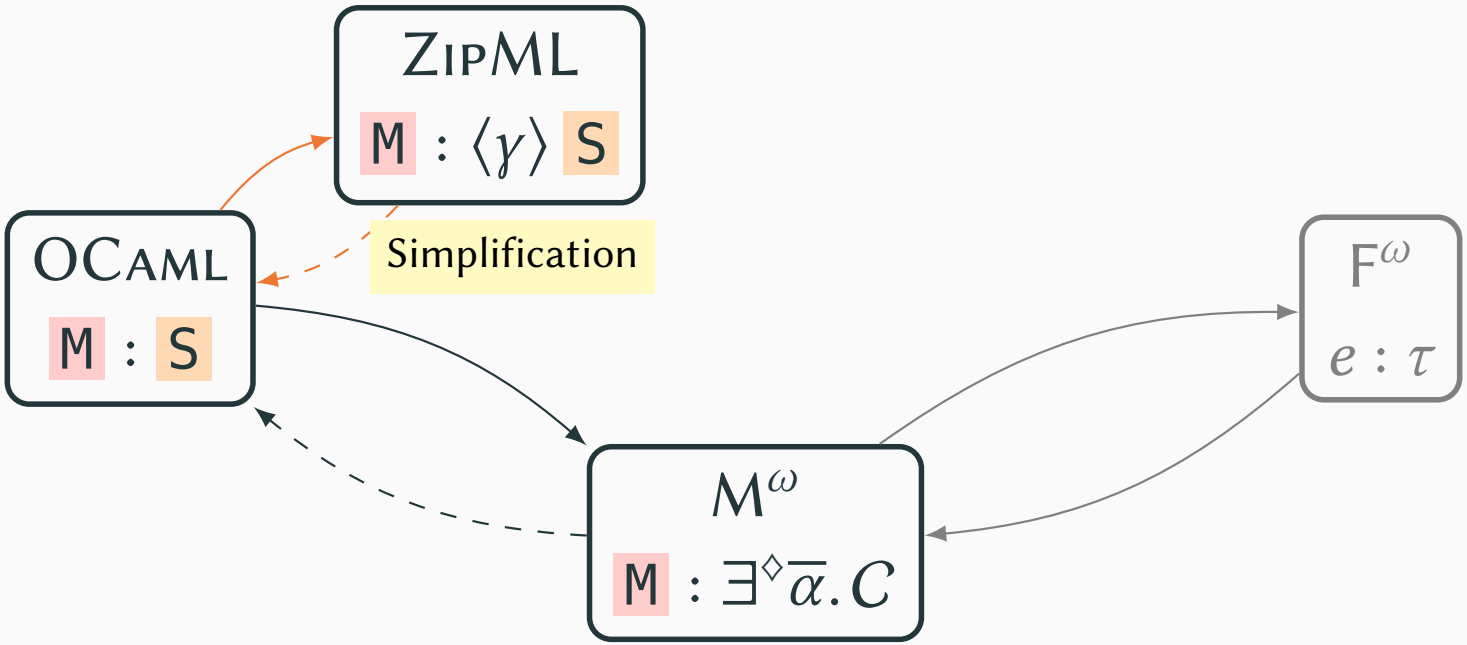

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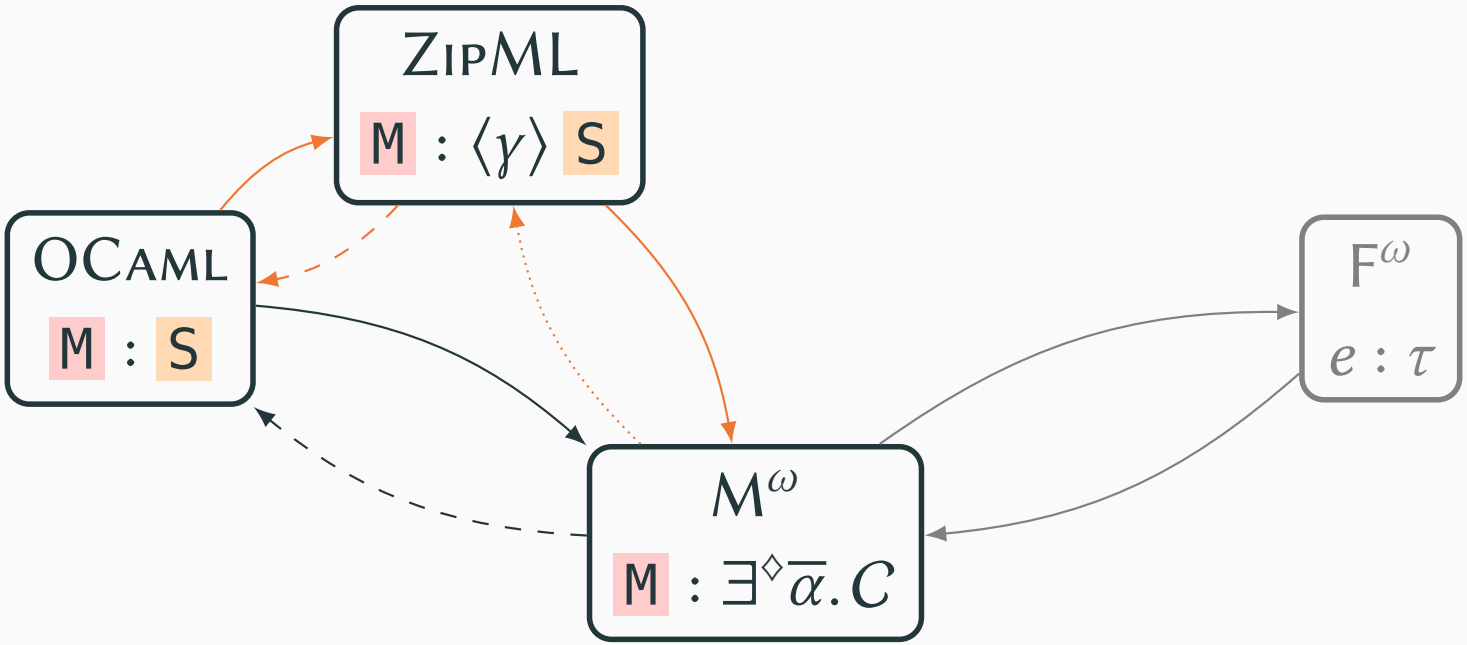

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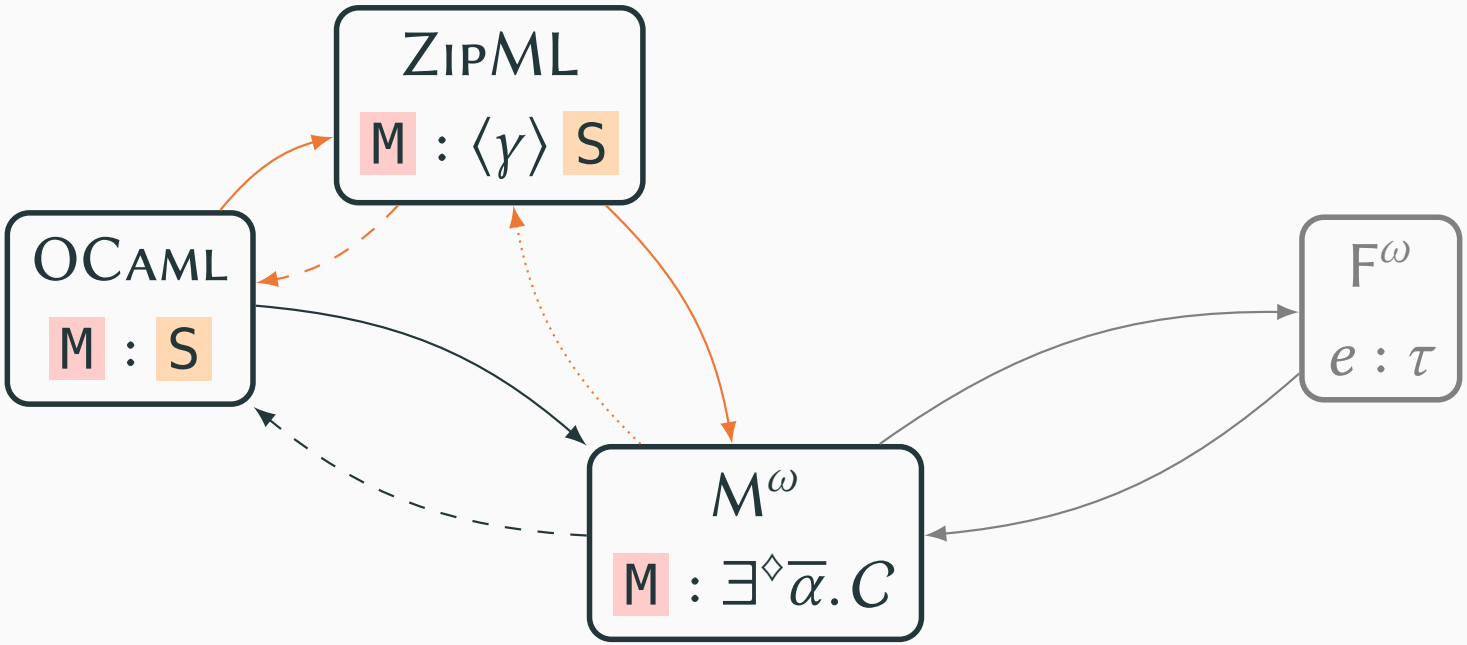

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- ✓ Module identities, transparent ascription, etc.

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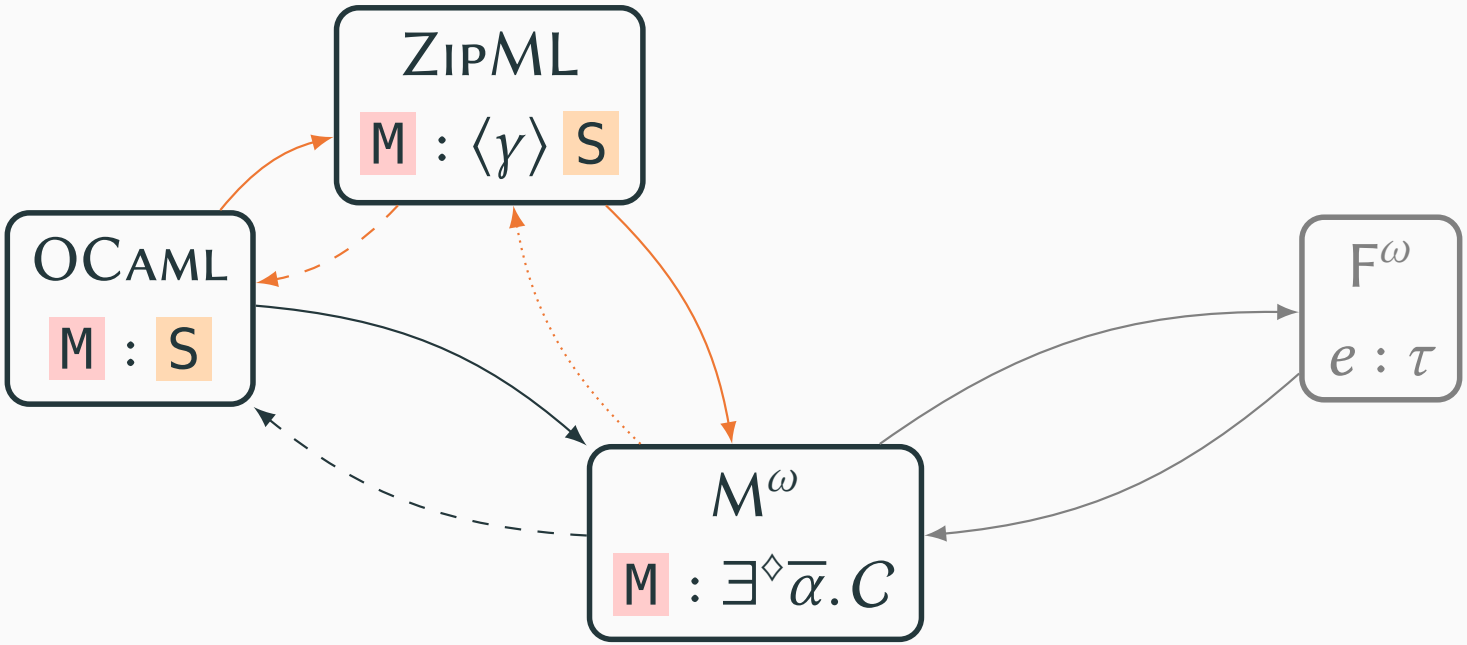

- ✓ Significant subset of OCaml
- ✓ Clear treatment of avoidance
- ✓ Module identities, transparent ascription, etc.
- ✓ User-defined names
- ✓ Lazyness
- ✓ Retrofit

Conclusion

Take-away

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$\mathbf{M}^{\omega}$

- A type system based on translation into F^{ω}
- Transparent existentials for sealing inside applicative functors

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OOPSLA 2024 - *Fulfilling OCaml modules with transparency*

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- Overview of the design space of module systems besides abstraction

Future works

Missing features

- The *easy* ones: **`include`**, **`open`**, 1st-class modules, **`module type of`**, signature modifiers (**`with type`**)
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- F^{ω} with kind functions and parametric kind polymorphism
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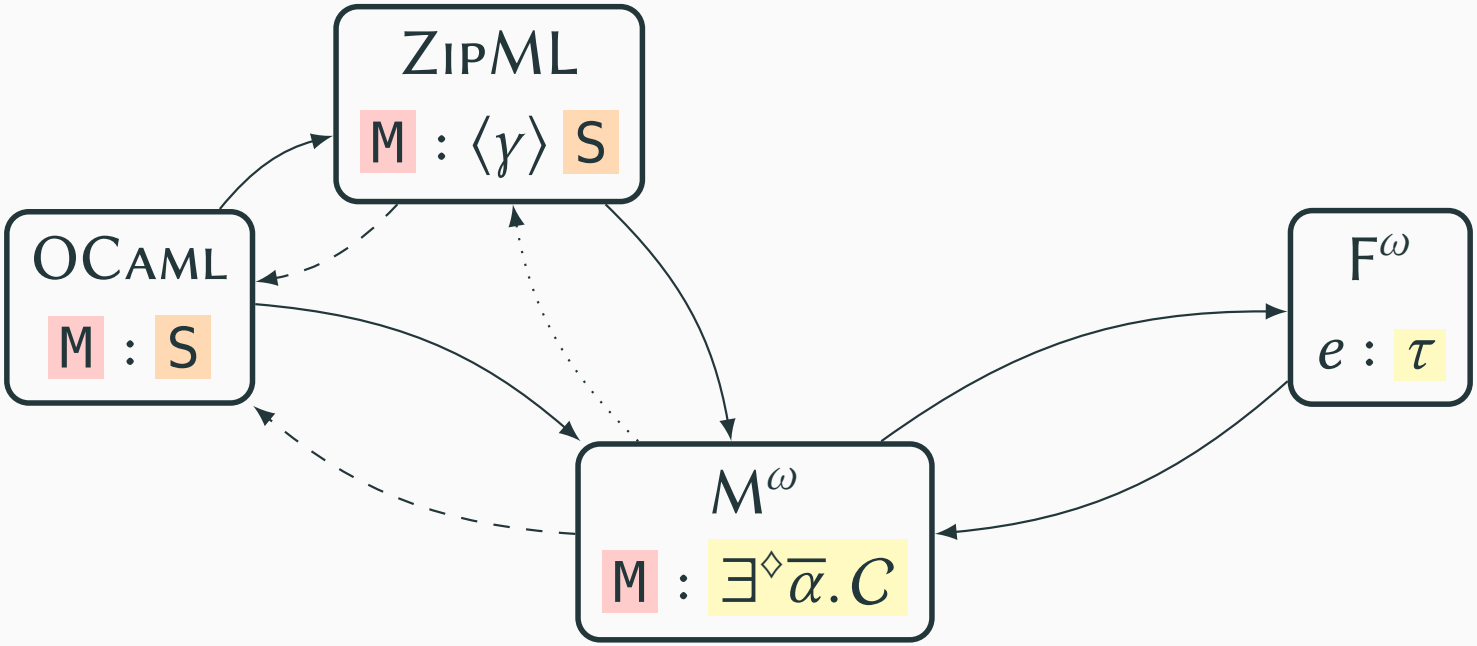
Research questions

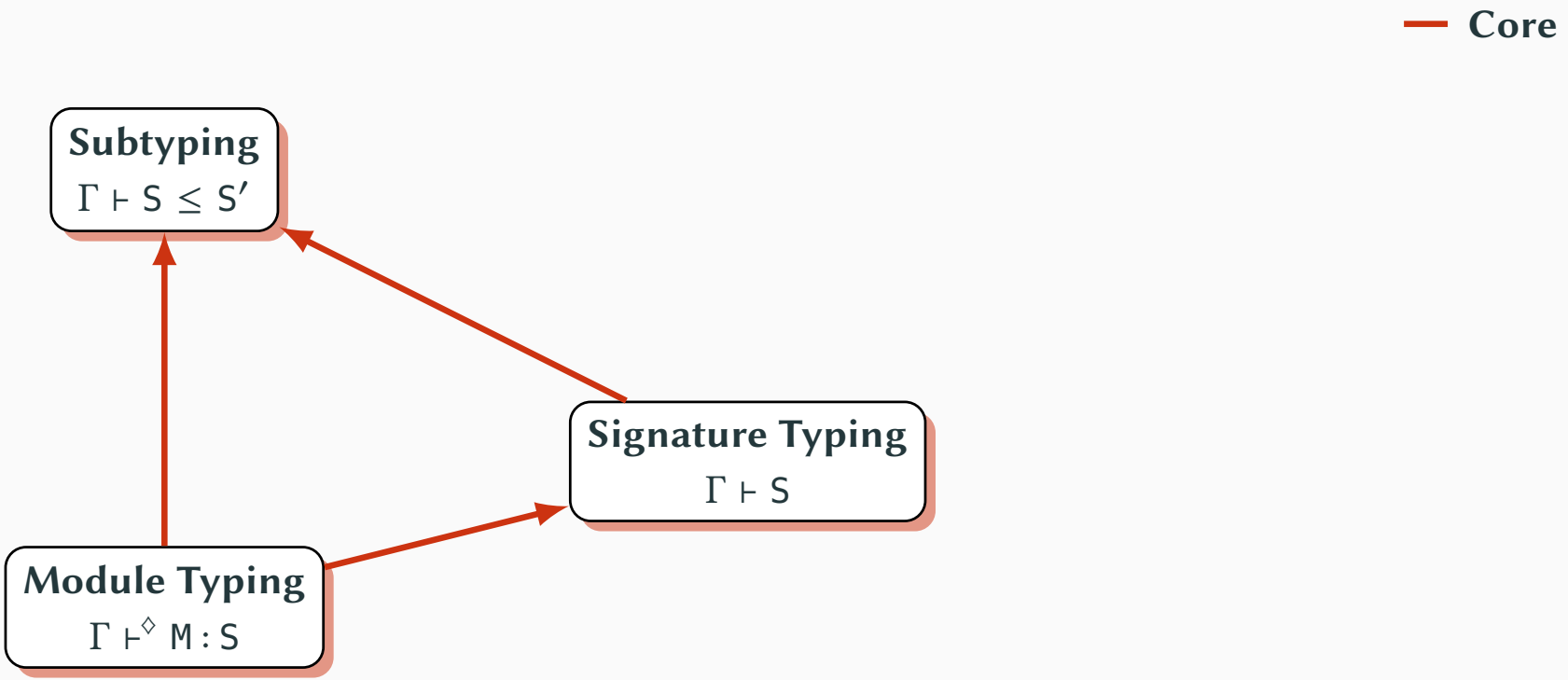
- More subtyping without loss of type-sharing
- Completeness of ZipML with full zippers
- Heuristics of zipper simplification

Questions ?



Backups

ZipML overview



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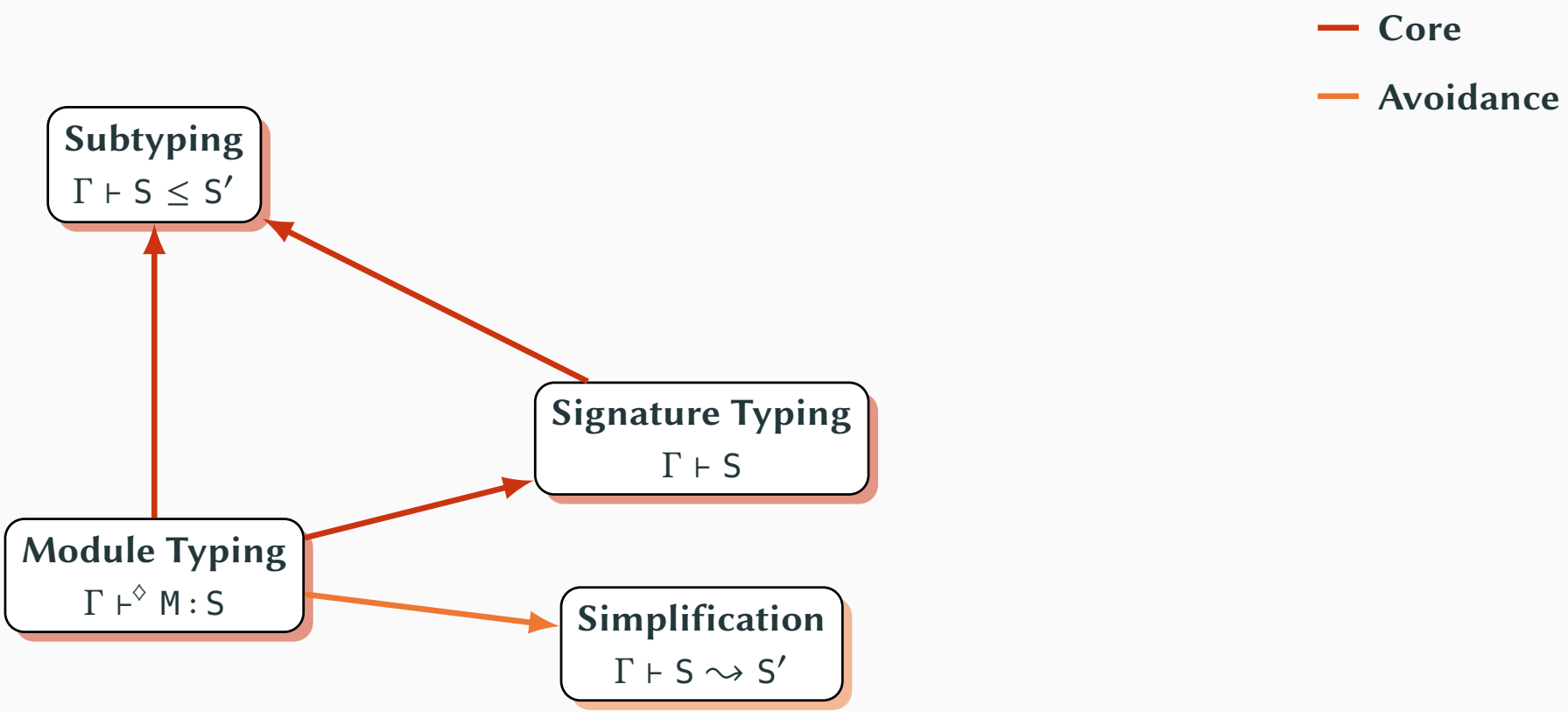

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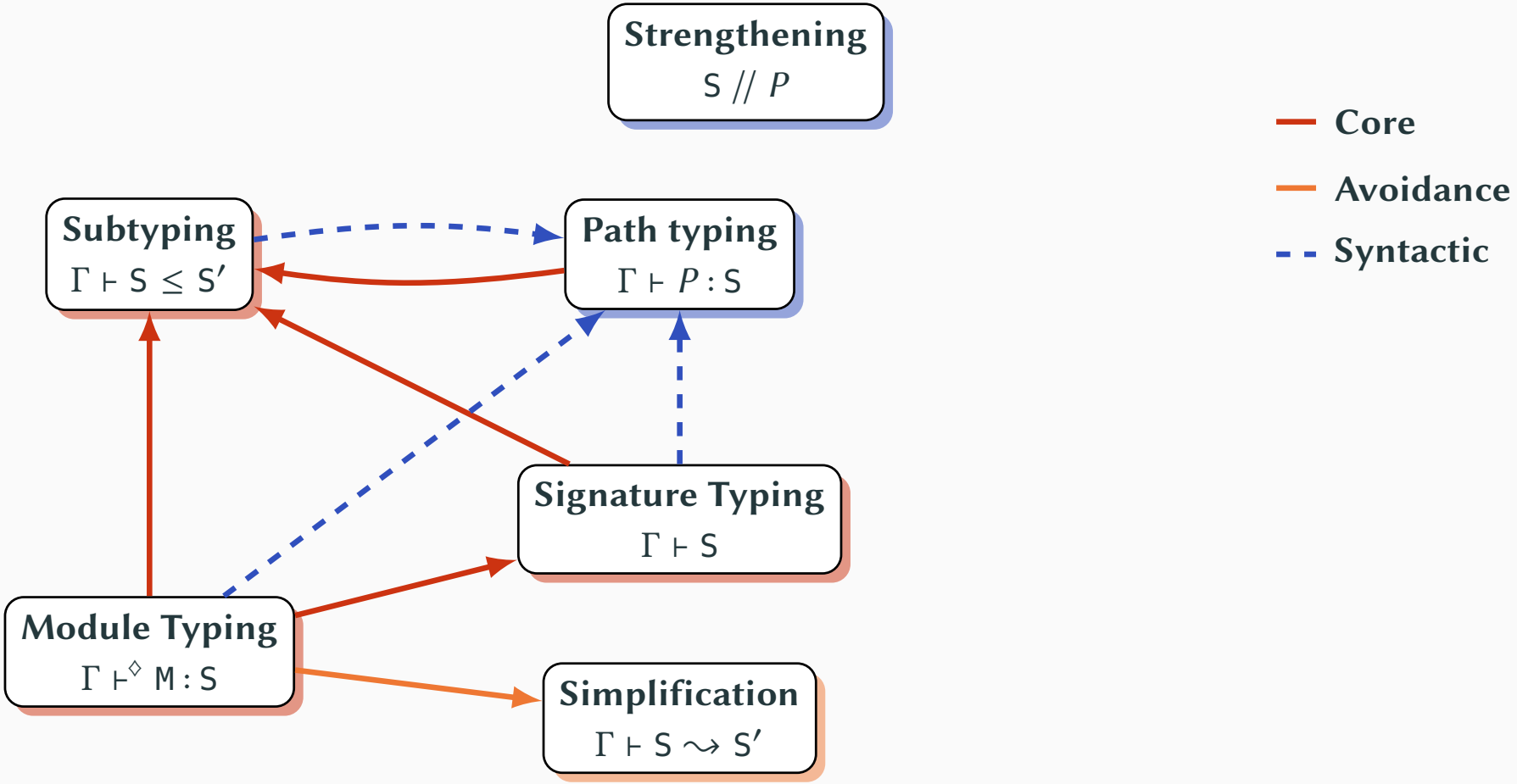

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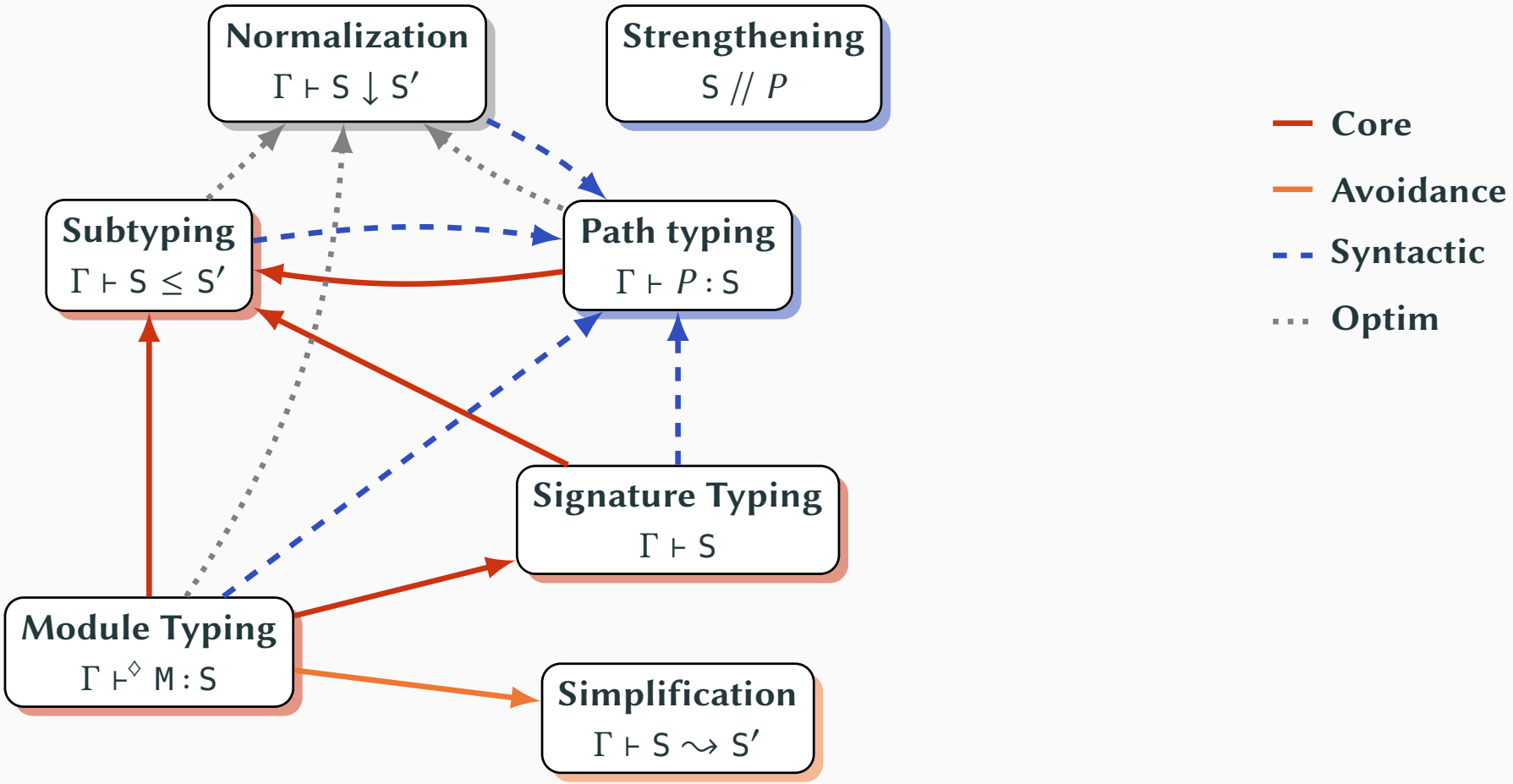

Strengthening in syntactic module systems

```
1 module X  = struct type t end
2 module X' = X
```

```
module X : sig type t end
```

```
module X' : ?
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$$\Gamma = (\texttt{X : sig type t end})$$

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Γ = (X : **sig type** t **end**)

```
module X' : sig type t end
X.t and X'.t are not equal
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$\longrightarrow$ Duplicating a signature may loose type-sharing

Strengthening in syntactic module systems

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- Need *strengthening* $\mathsf{S} /\!/ X$ to rewrite abstract type fields

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Strengthening in syntactic module systems

```
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2 module X' = X
```

```
module X : sig type t end
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Γ = `(X : sig type t end // X)`

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→ Rewriting big signatures is costly (duplication + traversal)
→ ZipML uses *lazy* strengthening

Module Magic

```
let module_magic : 'a → 'b =
  fun (x : 'a) →
    let module M = struct
      module rec N : sig val y : unit → 'b end = N
    end
    in M.N.y ()
```

```
val module_magic : 'a → 'b
```